Technologies and Models for En Route Commerce

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Definitions

- Electronic Commerce:

  Buying and selling goods using electronic transaction processing technologies (that require little or no intervention on the part of the buyer or seller).

- En Route Commerce:

  Electronic commerce conducted while traveling from one location to another.

Enroute Commerce for Personal Travel

- General Travel
  - Lodging
    - Pricing/discounts and availability
    - Reservations
    - Electronic check-in/keys
  - Restaurants and Entertainment
    - Information
    - Reservations
    - Electronic ticketing
- Automobile Travel
  - Gasoline pricing/purchases
  - Parking pricing/reservations/auctions
  - Roadside service

Enroute Commerce for Personal Travel (cont.)

- Air/Bus/Train Travel

  Airport Services
  Travel "aids" (e.g., movies)
  Porters
  Purchase rail tickets/passes/e-tickets

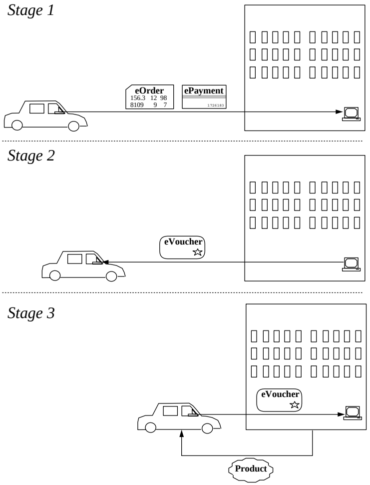
- Taxi Travel

  Competitive, dynamic pricing

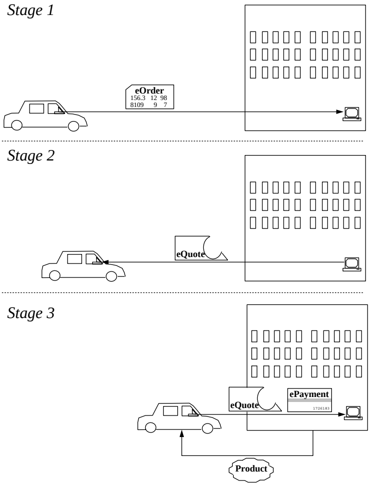
"Commercial" Enroute Commerce

- Mobile asset management by carrier
  - Load matching
  - Driver utilization
  - Equipment utilization
  - Dynamic re-routing
- Mobile asset management by shipper
  - Dynamic redployment of shipments
  - Improved responsiveness (info and controls)
- Personal services (as above)

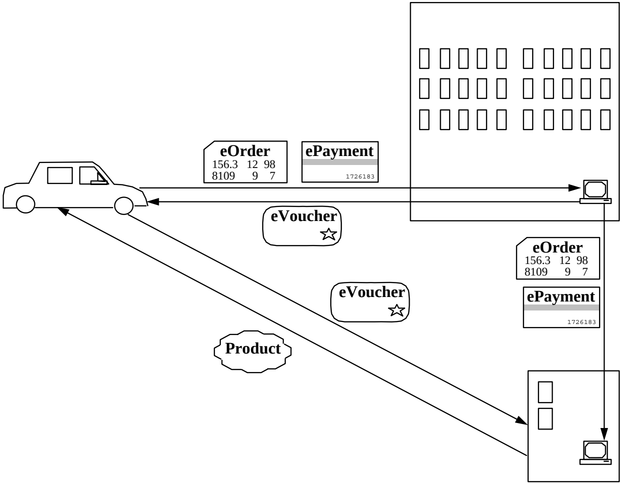
Type-1 Transactions



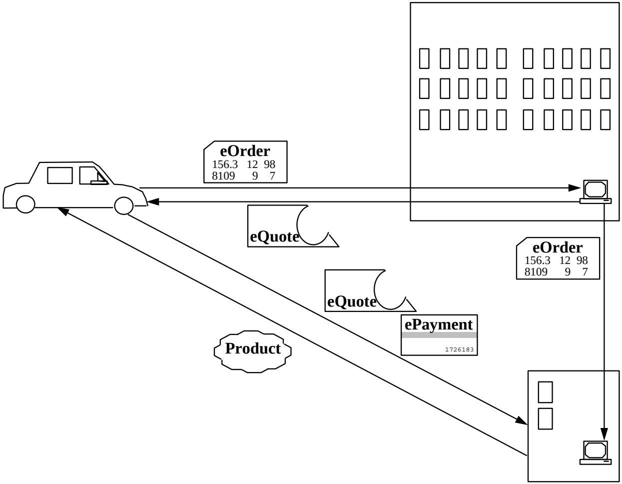
Type-2 Transactions



Type-3 Transactions (Using A Central Office)



Type-4 Transactions (Using A Central Office)



Taxi Services: Overview

- The Past:
  - Manual requests/reservations
  - Drop fee and per mile (e.g., NYC) or zone-based fares (e.g., DC)
  - Cash payments
- Today/The Near Future:
  - Electronic requests/reservations
  - Electronic payment
- The Future:
  - Real-time pricing?

Taxi Pricing: Assumptions

- Short, Frequent Auctions:

  Every 5 minutes

- Falling Price Auction:

  Firms announce prices for particular trips
  Individuals accept/reject
  Firms adjust prices
  ⋮
  All individuals receive “final” price

Modeling Assumptions

- Continuum of utility-maximizing passengers
- Continuum of homogeneous vehicles
- An available alternative (i.e., elastic demand)
- One time period/auction
- Small number of (or one) profit-maximizing firms
- One or more OD-pairs

Model

- Related Models:

  Spatial Price Equilibrium
  (Samuleson, 1952; Takayama and Judge, 1971)

  Spatial Oligopoly
  (Weskamp, 1985; Harker, 1986; Nagurney, 1987)

  Spatial Oligopoly with Arbitrageurs
  (Friesz and Bernstein, 1992)

- Important Difference:

  Customers can move

  "Product" can only be moved by the firm

Formulations

- Complementarity Problem:

  Given a function $G : R^n_+ \rightarrow R^n$, find an $x \in R^n$ such that:

$$\begin{array}{llll} x_i \geq 0 & G_i(x) \geq 0 & G_i(x)x_i = 0 & \forall i \in T_1 \\ x_i \text{ free} & G_i(x) = 0 & G_i(x)x_i = 0 & \forall i \in T_2, \end{array} \qquad (1)$$

  where $T_1 \cup T_2$ is a partition of the indices $\{1, 2, \cdots, n\}$.

- Variational Inequality Problem:

  Given a function $G : R^n_+ \rightarrow R^n$ and a (closed convex) set $X \subseteq R^n$, find an x such that:

$$G(x)^T(y - x) \geq 0 \ \forall y \in X. \qquad (2)$$

Existence and Uniqueness

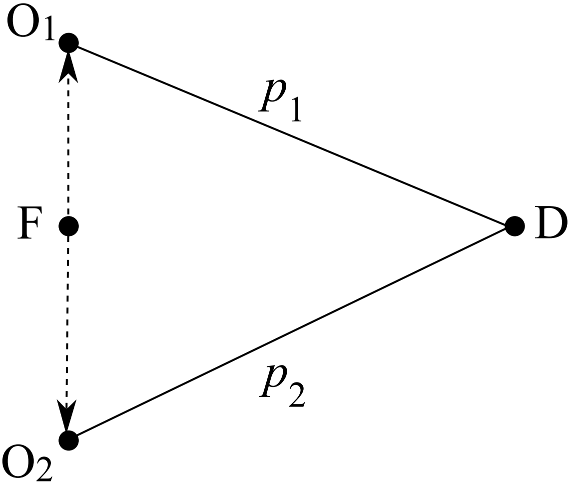
- Existence Conditions:

  Involve continuity

- Uniqueness Conditions:

  Involve some form of monotonicity

A Monopoly Example



A Monopoly Example (cont.)

- Demand:

$$p_1 = 4 - q_1$$

$$p_2 = 6 - 2q_2$$

- Marginal Cost:

$$C' = 0.2 \text{ for all trips}$$

A Monopoly Example (cont.)

- Prohibitvely High Walking Cost:

  $p_1 = 2.10$, $p_2 = 3.10$

  $q_1 = 1.90$, $q_2 = 1.45$

  $\pi_1 = 7.82$

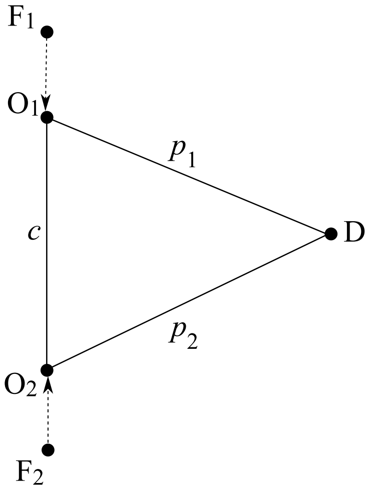
- Walking Cost of 0.5:

  $p_1 = 2.27$, $p_2 = 2.27$

  $q_1 = 1.73$, $q_2 = 1.62$

  $\pi_2 = 7.73$

An Oligopoly Example



An Oligopoly Example (cont.)

- Demand:

$$p_1 = 4 - q_1$$

$$p_2 = 13 - 2q_2$$

- Costs:

$$C'_{11} = 0.2$$

$$C'_{12} = 0.2 + c$$

$$C'_{21} = 2.1 + c$$

$$C'_{22} = 2.1$$

where $c = 3.0$

An Oligopoly Example (cont.)

- Prohibitively High Walking Cost:

  $p_1 = 2.10$, $p_2 = 6.10$

  $q_{11} = 1.90$, $q_{12} = 1.45$, $\pi = 7.82$

  $q_{21} = 0.00$, $q_{22} = 2.00$, $\pi = 8.00$

- Low Walking Cost:

  Firms can "allow" walkers

  Firms can "prevent" walkers (by pricing appropriately)

An Oligopoly Example (cont.)

- Assume:

  Walking cost of 3.0 (i.e., equal to c)

  Firms prevent walking

- One Solution:

  $p_1 = 2.40 \uparrow$, $p_2 = 5.40 \downarrow$

  $q_{11} = 1.60 \downarrow$, $q_{12} = 1.80 \uparrow$

  $q_{21} = 0.00$, $q_{22} = 2.00$

En Route Gasoline Purchases: Overview

- The Past:
  - Historical information about prices
  - Cash payments
- Today/The Near Future:
  - Location of some facilities
  - Real-time reported prices at some facilities
  - Electronic payment
- The Future:
  - Comprehensive real-time guaranteed prices
  - Decision support system that incorporates gasoline purchases

Assumptions

1. Given an origin and destination, the decision support system determines the best gas station and route.
2. There is only one gas station at any node. (We can still handle real-world situations by adding "dummy nodes".)
3. We only consider *short trips* (i.e., the amount of fuel in the vehicle is sufficient to reach all gas stations and the tank holds enough fuel to get to the destination).
4. The driver only wants to stop (for gasoline) once.
5. The driver's value of time is a known constant (α).
6. The driver always drives at exactly the speed limit.

Notation

O denotes the origin and D denotes the destination

$a_{ij} = 1$ if link j is directed out of node i, $a_{ij} = -1$ if link j is directed into node i, and $a_{ij} = 0$ otherwise

$b_O = 1$, $b_D = -1$, and $b_i = 0, i \in \mathcal{N} - \{O, D\}$

p_g denotes the price (per gallon) of gasoline at station g

m denotes the *fuel efficiency* (in miles per gallon) of the vehicle

s_{ij} denotes the speed limit on link (i, j)

c_{ij} denotes the total cost of traveling on link (i, j)

An Observation

- The Observation:

  We can ignore the gasoline in the vehicle at the start of the problem

- Different Rationales:

  It's a fixed cost

  Opportunity cost of the gas in the tank is the price you purchase gas at today

Link Costs

$$c_{ij} = p \cdot (d_{ij}/m) + \alpha \cdot (d_{ij}/s_{ij}) \tag{3}$$

An Algorithmic Approach

Use a multi-copy network in which there is a copy of the physical link for each gasoline price

Find the shortest path from O to D via the gas statione (i.e., through an intermediate node)

The Shortest Path (S.P.) Problem

$$\begin{array}{rrcl} \min_x & c^\top x & & \\ \text{s.t.} & Ax & = & b \\ & x & \in & \{0,1\}^n \end{array} \tag{4}$$

S.P. Through an Intermediate Node

Let h_I denote an n-vector in which all elements that that correspond to arcs with head node I (i.e., that are inbound to node I) have a value of 1 and all other elements have a value of 0.

$$
\begin{array}{rrcl}
\min_x & c^\top x & & \\
\text{s.t.} & Ax & = & b \\
 & x & \in & \{0,1\}^n \\
 & h_I^\top x & >= & 1
\end{array}
\tag{5}
$$

S.P. Through an Intermediate Node (cont.)

- A Well-Known Algorithm:

  Find the shortest path from O to I and the shortest path from I to D

- A Less-Well-Known Algorithm:

  Use a label setting algorithm and solve from I to O and D simultaneously (perhaps using two labels if $d_{ij} \neq d_{ji}$ for some i, j)

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S.P. Through an Intermediate Node (cont.)

Lagrangean Relaxation

$$\begin{array}{rrcl} \min_x & c^\top x + \lambda(1 - h_I^\top x) & & \\ \text{s.t.} & Ax & = & b \\ & x & \in & \{0,1\}^n \end{array} \tag{6}$$

S.P. Through an Intermediate Node (cont.)

The Shortcoming of Lagrangean Relaxation

$$\begin{array}{rrcl} \min_x & (c - \lambda h_I)^\top x + \lambda & & \\ \text{s.t.} & Ax & = & b \\ & x & \in & \{0,1\}^n \end{array} \tag{7}$$

λ has the effect of reducing the distance on the nodes that are inbound to I

λ must be so large that the important elements of $(d - \lambda h_I)$ are negative

SP Through One of a Set of Nodes

- Solve the k intermediate node problems and choosing the shortest
- Use $k+1$ labels on each node, one for the "unconstrained" distance and one for the distance through each of the intermediate nodes

Future Research on Real-Time Taxi Pricing

- Walking at the destination
- Taxi transfers (at both ends)
- Supply constraints
- Heterogeneous vehicles
- Multiple periods

Future Research on Side Trips

- Gasoline can be purchased on either the outbound or return trip
- The amount of fuel in the vehicle at the start of the trip constrains the choices
- More than one stop for fuel (and other things) must be made
- Other objectives (e.g., not diverting too far)
- Parallel processing (e.g., shared memory, vector processing)