# Congestion Pricing with an Untolled Alternative*

**David Bernstein**
Statistics and Operations Research Program
Princeton University
Princeton, NJ 08544, U.S.A.

**Ibrahim El Sanhouri**
Riyadh Development Authority
Riyadh, Saudi Arabia, 11614

April 1994

**Abstract**

Early research on congestion pricing took a very abstract view of demand. That is, it considered the demand for a transportation facility in a very aggregate fashion. Now, researchers are beginning to pay increasing attention to the choice processes that give rise to these demand functions (i.e., the decision to travel, origin/destination choice, mode choice, route choice, and departure-time choice). However, in so doing, past research has almost always assumed that all facilities will be tolled. That is, little attention has been given to the kind of facility-based pricing that is currently being proposed and tested in the United States (i.e., in which at least one alternative is left untolled. This paper demonstrates that it may be impossible to properly price more than one choice process when one alternative is left untolled.

## 1 Introduction

An interesting dichotomoy is becoming apparent in the application of congestion pricing. That is, two different "flavors" of congestion pricing are being applied in different parts of the world. In the United States the attention has been on facility-based pricing. For example, the pilot project in San Francisco and the various private toll road pricing projects that have been proposed are all facility-based.

*This draft contains a number of equations that are intended to make the results easier to verify. Unnecessary detail will be removed later.

On the other hand, in the rest of the world the attention has been primarily on area-based pricing. For example, the Singapore system charges for entry into the CBD during the morning peak (and for exit in the afternoon), the Hong Kong experiment subdivided the city districts into several zones and commuters were tolled when crossing zone-boundaries, the Norwegian cities of Oslo and Bergen constructed toll cordons around their CBD's and are poised to implement peak-period price differentials, and the planned schemes in London, Cambridge (UK), and Stockholm are all based on one form or another of area-wide pricing [see Hau (1992) for a review of these various programs].

Two arguments have been given for facility-based pricing. First, it is generally believed that it will be easier to get the public to accept congestion pricing if an alternative is left untolled [see, for example, the discussion in El Sanhouri (1994)]. Further, researchers have observed that such schemes can be optimal. In particular, in a static model of route pricing it is easy to show that it is possible to implement first-best tolls even when one of the routes connecting each origin-destination pair must be left untolled. This is because the desired changes in behavior are a result only of the difference in tolls, not their absolute level. Hence the toll on one facility can always be zero.

Unfortunately, such arguments ignore the fact that congestion pricing can, and should, be used to influence more than one type of choice process. Though traditional treatments of congestion pricing consider demand in the aggregate, transportation planners know that it is important to consider the different choice processes that constitute the demand "function". That is, the demand for a particular facility at a particular time results from the confluence of decisions to travel, origin/destination choices, mode choices, route choices, and departure-time choices.

In principle, tolls can and should be used to price all of these different choices, since each results in congestion (and hence is not priced at marginal cost). In other words, travel tolls should be used to influence the total number of trips, origin/destination tolls should be used to influence the starting and ending points of trips, mode-specific tolls should be used to influence mode splits, route (and/or link) tolls should be used to influence route splits, and time-varying tolls should be used to influence departure-time choices.

The purpose of this paper is to show that it may be impossible to properly price more than one choice process when one alternative must be left untolled for political reasons. To that end, this paper is related to the earlier work on suboptimal and/or second-best congestion pricing by Marchand (1968), Arnott (1979), Sullivan (1983), Wilson (1983), Braid (1987), and d'Ouville and McDonald (1990).

First, we will consider a situation in which we want to price both route choices and "mode" choices (i.e., we want to influence both route choices and the total number of auto commuters). We will show, using a simple two-route example, that it is impossible to achieve both the optimal mode splits and the optimal route splits when one route is left untolled. Second, we will consider a situation in which we want to price both route and departure-time choices. We will show using a simple two-route example, that it is impossible to achieve the ultimate route splits and the ultimate departure-time pattern when one route is left untolled. This result will be shown to hold for both continuously time-varying tolls and interval-based (i.e., step) tolls. We will conclude with a discussion of possible areas of future research.

## 2 Mode and Route Pricing

We begin by considering a situation in which only mode and route tolls are used to manage demand. For simplicity, we consider a network with one origin-destination pair and two non-overlapping highway routes.

First, let $\Re$, $\Re_+$, and $\Re_{++}$ denote the real numbers, non-negative reals, and positive reals respectively, and let $C_i : \Re_+ \rightarrow \Re_+$ denote the cost of travel on route $i$. We assume that the route cost functions are affine, that is:

$$C_i(N_i) = a_i + b_i N_i \tag{1}$$

where $a_i \in \Re_+$ is the free-flow travel time one route $i$, $b_i \in \Re_+$ represents the "congestion effect", and $N_i \in \Re_+$ denotes the total number of commuters using route $i$. Such route cost functions arise, for example, out of the equilibrium departure-time choices of commuters when congestion is caused by a deterministic bottleneck (as is discussed in more detail below). Without loss of generality, we assume that $a_1 < a_2$.

We also assume that commuters choose between driving and another mode. This mode choice process is further assumed to result in the following inverse mode choice function, $C_H : \Re_{++} \rightarrow \Re_+$:

$$C_H(M) = d - eM \tag{2}$$

where $M \in \Re_{++}$ denotes the number of highway users, and $d \in \Re_+$ and $e \in \Re_+$ are parameters. This mode choice process can be viewed as one particular type of (inverse) highway demand function.

Throughout this section, we will assume that the system is "well-behaved" in the following sense:

**Definition 2.1** *A two-route network is said to be* regular *iff:*

$$a_j > a_i \Rightarrow M \geq \frac{a_j - a_i}{b_i} \tag{3}$$

*for $j \neq i$, and*

$$d > \frac{a_1 b_2 + a_2 b_1}{b_1 + b_2}. \tag{4}$$

Regularity condition (3) simply ensures that both routes are used in equilibrium (which is the only case of any interest). To see this, observe that:

$$M \geq \frac{a_j - a_i}{b_i} \quad \Leftrightarrow \quad a_i + b_i M \geq a_j \tag{5}$$

$$\Leftrightarrow \quad C_i(M) \geq a_j. \tag{6}$$

So, regularity condition (3) requires that $a_i < a_j \Rightarrow C_i(M) \geq a_j$ which does, in fact, ensure that both routes are used. Similarly, regularity condition (4) ensures that both modes are used in equilibrium. This follows from (2) and (10) below.

Given this regularity assumption, we know that the equilibrium route split, $\overline{N} = (\overline{N}_1, \overline{N}_2) \in \Re^2_+$, must satisfy $C_1(\overline{N}_1) = C_2(\overline{N}_2)$. Hence,

$$C_1(\overline{N}_1) = C_2(\overline{N}_2) \quad \Rightarrow \quad a_1 + b_1 \overline{N}_1 = a_2 + b_2 \overline{N}_2 \tag{7}$$

$$\Rightarrow \quad a_1 + b_1 \overline{N}_1 = a_2 + b_2 M - b_2 \overline{N}_1 \tag{8}$$

$$\Rightarrow \quad \overline{N}_1 (b_1 + b_2) = a_2 - a_1 + b_2 M \tag{9}$$

$$\Rightarrow \quad \overline{N}_1 = \frac{a_2 - b_1}{b_1 + b_2} + \frac{b_2}{b_1 + b_2} M. \tag{10}$$

Further, letting $\mathcal{C}_i(N_i) = C_i(N_i) N_i$, we know that the optimal route split must satisfy $\frac{\partial \mathcal{C}_1}{N_1} = \frac{\partial \mathcal{C}_2}{N_2}$. Hence,

$$\left.\frac{\partial \mathcal{C}_1}{\partial N_1}\right|_{N_1^*} = \left.\frac{\partial \mathcal{C}_2}{\partial N_2}\right|_{N_2^*} \quad \Rightarrow \quad a_1 + 2b_1 N_1^* = a_2 + 2b_2(M - N_1^*) \tag{11}$$

$$\Rightarrow \quad N_1^* 2(b_1 + b_2) = (a_2 - a_1) + 2b_2 M \tag{12}$$

$$\Rightarrow \quad N_1^* = \frac{(a_2 - a_1)}{2(b_1 + b_2)} + \frac{b_2}{b_1 + b_2} M. \tag{13}$$

Now, there are two different ways to achieve the optimal route split using tolls. In the first, both routes are tolled with the toll on route $i$, $\mu_i \in \Re_{++}$, given by:

$$\mu_i = \left.\frac{\partial \mathcal{C}_i}{\partial N_i}\right|_{N_i^*} - C_i(N_i^*). \tag{14}$$

Hence, the cost that commuters incur when both routes are tolled, $C^{\mu} : \Re_{++} \rightarrow \Re_{++}$, is given by:

$$
\begin{aligned}
C^{\mu}(M) &= C_1(N_1^*) + \mu_1 = C_2(N_2^*) + \mu_2 = \left.\frac{\partial \mathcal{C}_1}{\partial N_1}\right|_{N_1^*} = \left.\frac{\partial \mathcal{C}_2}{\partial N_2}\right|_{N_2^*} && (15)\\
&= a_1 + \frac{b_1(a_2 - a_1)}{b_1 + b_2} + \frac{2b_1b_2}{b_1 + b_2}M && (16)\\
&= \frac{a_1b_1 + a_1b_2 + a_2b_1 - a_1b_1}{b_1 + b_2} + \frac{2b_1b_2}{b_1 + b_2}M && (17)\\
&= \frac{a_1b_2 + a_2b_1}{b_1 + b_2} + \frac{2b_1b_2}{b_1 + b_2}M. && (18)
\end{aligned}
$$

Alternatively, since it is only the difference in the two tolls that affects the route splits, it is also possible to achieve the optimal route splits by tolling only route 1. In particular, given that $a_1 < a_2$, it follows that $a_1 < a_2 \Rightarrow \frac{a_2 - a_1}{2(b_1+b_2)} < \frac{a_2 - a_1}{b_1 + b_2}$ and hence that $\overline{N}_1 > N_1^*$. Therefore, assuming that we can only implement non-negative tolls, the toll must be placed on route 1 in order to reduce $\overline{N}_1$ to $N_1^*$. The value of this toll, $\sigma \in \Re_{++}$, is given by:

$$\sigma = \mu_1 - \mu_2. \tag{19}$$

Hence, since $\left.\frac{\partial \mathcal{C}_1}{\partial N_1}\right|_{N_1^*} = \left.\frac{\partial \mathcal{C}_2}{\partial N_2}\right|_{N_2^*}$, it follows that:

$$\sigma = C_1(N_1^*) - C_2(N_2^*). \tag{20}$$

The cost incurred by commuters in the presence of the optimal on-route toll, $C^{\sigma} : \Re_{++} \rightarrow \Re_{++}$, is given by:

$$
\begin{aligned}
C^{\sigma}(M) &= C_2(N_2^*) = C_1(N_1^*) = a_2 + b_2N_2^* && (21)\\
&= a_2 + \frac{b_2(a_1 - a_2)}{2(b_1 + b_2)} + \frac{b_1b_2}{b_1 + b_2}M. && (22)
\end{aligned}
$$

Hence, we get the following (well-known) result:

**Lemma 2.1** *For a regular two-route network, the cost incurred by commuters in the presence of the optimal one-route toll, $\sigma$, on route 1 is less than the cost incurred when the optimal two-route toll, $\mu = (\mu_1, \mu_2)$, is in place. That is, $C^{\sigma}(M) < C^{\mu}(M)$.*

**Proof.** Since $N_2^*$ is identical in both cases, the result follows immediately from the fact that $C_2(N_2^*) = a_2 + b_2 N_2^* < a_2 + 2b_2 N_2^* = \frac{\partial C_2}{\partial N_2}\Big|_{N_2^*}$. Q.E.D.

Of course, the total social cost, $\mathcal{S} : \Re_+^2 \to \Re_{++}$, is identical in both cases and given by:

$$\mathcal{S} = C_1(N_1^*)N_1^* + c_2(N_2^*)N_2^*. \tag{23}$$

Given this, without even solving for the optimal route split and highway usage, it is possible to demonstrate the following:

**Theorem 2.1** *It is not possible to achieve the optimal mode split on a regular network when route 2 cannot be tolled and the optimal one-route toll, $\sigma$, is in place on route 1.*

**Proof.** Since we know from Lemma 2.1 that the cost incurred by commuters (on both routes) is lower when only route 1 is tolled, we need only show that the inverse mode split function evaluated at the optimal number of highway commuters, $M^*$, is larger than the cost incurred when both routes are tolled.

To do so, observe that:

$$\begin{aligned}
\frac{\partial \mathcal{S}}{\partial M} &= \frac{\partial \mathcal{C}_1(N_1^*)}{\partial M} + \frac{\partial \mathcal{C}_2(N_2^*)}{\partial M} & (24)\\
&= \frac{\partial \mathcal{C}_1(N_1^*)}{\partial N_1^*}\frac{\partial N_1^*}{\partial M} + \frac{\partial \mathcal{C}_2(N_2^*)}{\partial N_2^*}\frac{\partial N_2^*}{\partial M}. & (25)
\end{aligned}$$

But, since $\frac{\partial \mathcal{C}_1(N_1^*)}{\partial N_1^*} = \frac{\partial \mathcal{C}_2(N_2^*)}{\partial N_2^*}$ and $\frac{\partial N_1^*}{\partial M} + \frac{\partial N_2^*}{\partial M} = 1$ it follows that:

$$\frac{\partial \mathcal{S}}{\partial M} = \frac{\partial \mathcal{C}_1(N_1^*)}{\partial N_1^*}. \tag{26}$$

Now, observe that the optimal number of highway users, $M^*$, satisfies:

$$\frac{\partial \mathcal{S}}{\partial M}\Big|_{M^*} = d - eM^*. \tag{27}$$

Hence:

$$\frac{\partial \mathcal{S}}{\partial M}\Big|_{M^*} = d - eM^* = \mathcal{C}^\mu(M^*). \tag{28}$$

But, since $C^\mu(M^*) > C^\sigma(M^*)$ it follows that $d - eM^* > C^\sigma(M^*)$. Hence, the highway cost which yields the optimal mode split is large than the cost realized with the optimal one-route toll and the result follows. Q.E.D.

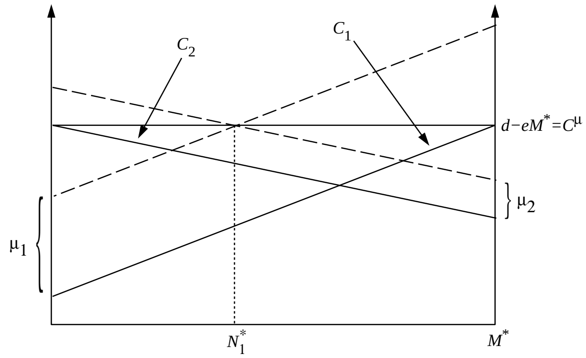


Figure 1: Tolling Both Routes

This result is illustrated in Figures 1, 2, and 3. As shown in Figure 1, we can toll both routes to achieve the optimal route and mode splits. However, given the optimal mode choices, Figure 2 shows that it is impossible to toll just route 1 and achieve the appropriate route splits. Another way to see this is with Figure 3. Here, given the optimal number of highway users, it is possible to get the optimal route splits. But, the resulting cost is inconsistent with (i.e., lower than) the inverse mode choice function.

Hence, we see that it is impossible to place a (non-negative) toll only on route 1 and achieve both the optimal route split and the optimal mode split.

# 3 Route and Departure-Time Pricing

We now turn our attention to the situation in which tolls are used to influence only the route and departure-time choices of commuters. Throughout this section we will use the model introduced by Vickrey (1969) and extended by Braid (1989), Arnott et al. (1990a, 1990b) and others. We assume that the *travel time* on each route is a function of the free-flow travel time and the delays caused by a bottleneck at the downstream end of the route. In particular, the travel time on route $i$ for vehicles departing at time $t$ is assumed to be given by:

$$R_i(t) = T_i^f + D_i(t + T_i^f)/s_i \tag{29}$$

where $T_i^f \in \Re_{++}$ denotes the free-flow travel time on route $i$, $D_i(t + T_i^f) \in \Re_+$ denotes the number of vehicles in the queue at time $t + T_i^f$, and $s_i \in \Re_{++}$ denotes the service rate of the deterministic queue on route $i$.

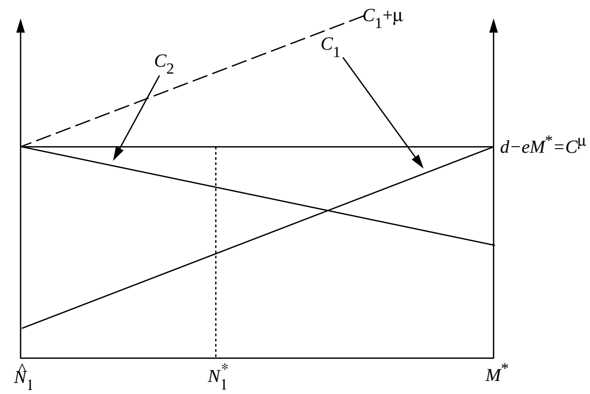


Figure 2: Putting a Toll Only on Route 1

Further, because travelers may arrive early or late, we introduce an asymmetric *schedule cost* given by:

$$\Phi_i(t) = \begin{cases} \beta[t^* - (t + R_i(t))] & \text{if} \quad t^* > [t + R_i(t)] \\ 0 & \text{if} \quad t^* = [t + R_i(t)] \\ \gamma[(t + R_i(t)) - t^*] & \text{if} \quad t^* < [t + R_i(t)], \end{cases} \tag{30}$$

where $\beta \in \Re_{++}$ denotes the dollar cost of early arrival time, $\gamma \in \Re_{++}$ denotes the dollar cost of late arrival time, and $t^*$ denotes the desired arrival time. Thus, the user cost of travel on route $i$ for vehicles departing at time $t$ is given by:

$$C_i(t) = \alpha R_i(t) + \Phi_i(t) \tag{31}$$

where $\alpha \in \Re_{++}$ denotes the dollar cost of travel time, We will assume throughout the discussion that $\beta < \alpha < \gamma$. Also, to simplify the notation somewhat, we will let

$$\delta = \frac{\beta\gamma}{\beta + \gamma}. \tag{32}$$

Given this, it is shown in Arnott et al. (1990b) [as an extension of the results in Vickrey (1969) and Arnott et al. (1990a)] that the equilibrium departure rate function for route $i$ is given by:

$$\overline{r}_i(t) \begin{cases} s_i + \frac{\beta s_i}{\alpha - \beta} & , t \in [t_{iq}, \tilde{t}_i) \\ s_i - \frac{\gamma s_i}{\alpha + \gamma} & , t \in (\tilde{t}, t_{iq'}] \end{cases} \tag{33}$$

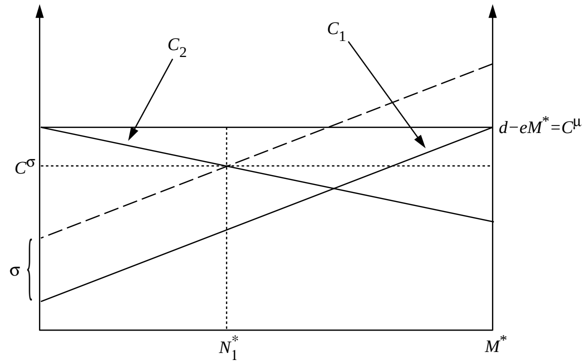


Figure 3: Another Look at Tolling Only Route 1

where $t_{iq}$ and $t_{iq'}$ are the times at which the departures begin and end, respectively, and $\tilde{t}$ is the departure time that results in an on-time arrival. On the other hand, the optimal departure rate function is given by:

$$r_i^*(t) = s_i \qquad , t \in [t_{iq}, t_{iq'}]. \tag{34}$$

The "critical times" are given by:

$$t_{iq} = t^* - \left(\frac{\gamma}{\beta+\gamma}\right)\left(\frac{N_i}{s_i}\right) - T_i^f \tag{35}$$

$$t_{iq'} = t^* + \left(\frac{\beta}{\beta+\gamma}\right)\left(\frac{N_i}{s_i}\right) - T_i^f \tag{36}$$

$$\tilde{t}_i = t^* - \left(\frac{\beta\gamma}{\alpha(\beta+\gamma)}\right)\left(\frac{N_i}{s_i}\right) - T_i^f \tag{37}$$

where $N_i$ denotes the total number of vehicles on route $i$.

### 3.1 Using a Continuously Time-Varying Toll

As shown in Vickrey (1969) and Arnott et al. (1990a,1990b), it is possible to impose a continuously time-varying toll that will result in people voluntarily choosing the optimal departure-times. This toll (when restricted to being non-negative) is given by:

$$\tau_i(t) = \begin{cases} \frac{\beta\gamma}{(\beta+\gamma)}\frac{N_i}{s_i} - \beta(\tilde{t} - t) & t \in [t_{iq}, \tilde{t}) \\ \frac{\beta\gamma}{(\beta+\gamma)}\frac{N_i}{s_i} - \gamma(t - \tilde{t}) & t \in [\tilde{t}, t_{iq'}] \end{cases} \tag{38}$$

That is, the toll at a particular departure time is simply the difference between the equilibrium and optimal cost at that time. As shown in Arnott et al. (1990b, Theorem 2), the equilibrium route choices in the presence of this departure-time toll on both routes will also be optimal.

We now consider the case in which only route $T$ is tolled, and route $U$ is left untolled. In this case, it follows from (35) that the total private cost on route $T$, $\mathcal{C}_T : \Re_+ \to \Re_{++}$ is given by:

$$\mathcal{C}_T(N_T) = \alpha T_T^f N_T + \delta \frac{N_T^2}{s_T} \tag{39}$$

while it follows from (33), (35), and (38) that the total social cost on route $T$, $\mathcal{S}_T : \Re_+ \to \Re_{++}$, is given by:

$$\mathcal{S}_T(N_T) = \alpha T_T^f N_T + \delta \frac{N_T^2}{2s_T}. \tag{40}$$

which is just the private cost minus the toll revenues. Since route $U$ does not have a departure-time toll, it follows from (35) that the total private cost, $\mathcal{C}_U : \Re_+ \to \Re_{++}$, is given by:

$$\mathcal{C}_U(N_U) = \alpha T_U^f N_U + \delta \frac{N_U^2}{s_U}. \tag{41}$$

We will, again, restrict our attention to regular networks. In this case:

**Definition 3.1** *A two-route network is said to be* regular *iff:*

$$T_U^f > T_T^f \Rightarrow M > \frac{\alpha}{\delta} s_T (T_U^f - T_T^f). \tag{42}$$

*and*

$$T_T^f > T_U^f \Rightarrow M > \frac{\alpha}{\delta} s_U (T_T^f - T_U^f). \tag{43}$$

Again, these regularity conditions ensure that both routes are used in equilibrium.

In order to determine the optimal route split we must minimize $\mathcal{T} = \mathcal{S}_T + \mathcal{C}_U$ subject to the constraint that $M = N_T + N_U$. Substituting for $N_U$ yields the following problem in one variable ($N_T$):

$$\min \mathcal{T} = \alpha T_T^f N_T + \delta \frac{N_T^2}{2s_T} + \alpha T_U^f (M - N_T) + \delta \frac{(M - N_T)^2}{s_U}. \tag{44}$$

Differentiating and solving yields:

$$\begin{aligned}
\frac{\partial \mathcal{T}}{\partial N_T} = 0 \quad &\Rightarrow \quad \alpha(T_T^f - T_U^f) + \delta \frac{N_T^*}{s_T} - \delta \frac{2M}{s_U} \delta \frac{N_T^*}{s_U} = 0 &(45)\\
&\Rightarrow \quad \delta N_T^* \left( \frac{1}{s_T} + \frac{2}{s_U} \right) = \delta \frac{2M}{s_U} - \alpha (T_T^f - T_U^f) &(46)\\
&\Rightarrow \quad N_T^* \left( \frac{s_U + 2s_T}{s_T s_U} \right) = \frac{2M}{s_U} - \frac{\alpha}{\delta} (T_T^f - T_U^f) &(47)\\
&\Rightarrow \quad N_T^* = 2M \left( \frac{s_T}{s_U + 2s_T} \right) - \frac{\alpha}{\delta} \left( \frac{s_T s_U}{s_U + 2s_T} \right) (T_T^f - T_U^f). &(48)
\end{aligned}$$

$$N_T^* = 2M \left( \frac{s_T}{s_U + 2s_T} \right) + \frac{\alpha}{\delta} \left( \frac{s_T s_U}{s_U + 2s_T} \right) (T_U^f - T_T^f). \tag{49}$$

On the other hand, the equilibrium route split can be determined by setting $C_T = C_U$ and solving for $\overline{N}_T$ as follows:

$$\begin{aligned}
\alpha T_T^f + \delta \frac{\overline{N}_T}{s_T} = \alpha T_U^f + \delta \frac{\overline{N}_U}{s_U} \quad &\Rightarrow \quad \alpha T_T^f + \delta \frac{M - \overline{N}_U}{s_T} = \alpha T_U^f + \delta \frac{\overline{N}_U}{s_U} &(50)\\
&\Rightarrow \quad \delta \left( \frac{\overline{N}_T}{s_T} + \frac{\overline{N}_T}{s_U} \right) = \alpha (T_U^f - T_T^f) + \delta \frac{M}{s_U} &(51)\\
&\Rightarrow \quad \delta \overline{N}_T \left( \frac{s_U + s_T}{s_U s_T} \right) = \alpha (T_U^f - T_T^f) + \delta \frac{M}{s_U}. &(52)
\end{aligned}$$

Hence:

$$\overline{N}_T = \frac{\alpha}{\delta} + \left( \frac{s_U s_T}{s_U + s_T} \right) (T_U^f - T_T^f) + \left( \frac{s_T}{s_U + s_T} \right) M. \tag{53}$$

Given this, we have the following important result:

**Lemma 3.1** *On a regular network with the optimal continuously time-varying toll on route $T$, the number of users of route $T$ in equilibrium is less than the optimal number (i.e., $\overline{N}_T < N_T^*$).*

**Proof.** Observe that $\overline{N}_T < N_T^*$ is equivalent to:

$$\begin{aligned}
\overline{N}_T \quad &= \quad \frac{\alpha}{\delta} + \left( \frac{s_U s_T}{s_U + s_T} \right) (T_U^f - T_T^f) + \left( \frac{s_T}{s_U + s_T} \right) M &(54)\\
&< \quad N_T^* = 2M \left( \frac{s_T}{s_U + 2s_T} \right) + \frac{\alpha}{\delta} \left( \frac{s_T s_U}{s_U + 2s_T} \right) (T_U^f - T_T^f) &(55)
\end{aligned}$$

Hence:

$$\overline{N}_T < N_T^* \Leftrightarrow (T_U^f - T_T^f)\frac{\alpha}{\delta}s_T\left[\frac{s_U}{s_U+s_T} - \frac{s_U}{s_U+2s_T}\right] < M\left[\frac{2s_T}{s_U+s_T} - \frac{s_T}{s_U+s_T}\right]. \tag{56}$$

Further, since:

$$\begin{aligned}
\frac{2s_T}{s_U+2s_T} - \frac{s_T}{s_U+s_T} &= \frac{2s_T(s_U+s_T) - s_T(s_U+2s_T)}{(s_U+2s_T)(s_U+s_T)} && (57)\\
&= \frac{2s_Ts_U + 2s_T^2 - s_Ts_U - 2s_T^2}{(s_U+2s_T)(s_U+s_T)} && (58)\\
&= \frac{s_Ts_U}{(s_U+2s_T)(s_U+s_T)}. && (59)
\end{aligned}$$

and

$$\begin{aligned}
\frac{s_U}{s_U+s_T} - \frac{s_U}{s_U+2s_T} &= \frac{s_U(s_U+2s_T) - s_U(s_U+s_T)}{(s_U+2s_T)(s_U+s_T)} && (60)\\
&= \frac{-s_U^2 - s_Us_T + s_U^2 + 2s_Us_T}{(s_U+2s_T)(s_U+s_T)} && (61)\\
&= \frac{s_Ts_U}{(s_U+2s_T)(s_U+s_T)} && (62)
\end{aligned}$$

it follows that:

$$\begin{aligned}
\overline{N}_T < N_T^* &\Leftrightarrow (T_U^f - T_T^f)\frac{\alpha}{\delta}s_T\left[\frac{s_Ts_U}{(s_U+2s_T)(s_U+s_T)}\right] < M\left[\frac{s_Ts_U}{(s_U+2s_T)(s_U+s_T)}\right] && (63)\\
&\Leftrightarrow (T_U^f - T_T^f)\frac{\alpha}{\delta}s_T < M. && (64)
\end{aligned}$$

Now, when $T_T^f \geq T_U^f$ this last inequality clearly holds. Hence, all that remains is to show that this inequality also holds when $T_T^f < T_U^f$.

To do so, observe from the regularity assumption that $T_T^f < T_U^f \Rightarrow M > (T_U^f - T_T^f)\frac{\alpha}{\delta}s_T$ Hence, the result follows. Q.E.D.

Thus, we immediately have the following:

**Theorem 3.1** *On a regular network with the optimal continuously time-varying toll on route $T$, it is impossible to achieve the optimal route-split with a non-negative route toll (i.e., with a non-negative uniform toll).*

**Proof.** Since the only way to increase the equilibrium number of users of $T$ is to make $T$ relatively less expensive, the result follows immediately from Lemma 3.1 and our inability to toll route $U$. Q.E.D.

In fact, it is possible to show that we cannot improve the route choices at all using a positive uniform toll. This is an immediate consequence of the following result:

**Theorem 3.2** *On a regular network with the optimal continuously time-varying toll on route $T$, the total social cost is decreasing in $N_T$ at the equilibrium $\overline{N}_T$ (i.e., $\frac{\partial \mathcal{T}}{\partial N_T}\Big|_{\overline{N}_T} < 0$).*

**Proof.** Observe from (44) that

$$\frac{\partial \mathcal{T}}{\partial N_T} = \alpha(T_T^f - T_U^f) + \delta N_T \left(\frac{s_U + 2s_T}{s_U s_T}\right) - \delta \frac{2M}{s_U}. \tag{65}$$

Hence,

$$\begin{aligned}
\frac{\partial \mathcal{T}}{\partial N_T}\Big|_{\overline{N}_T} &= \alpha(T_T^f - T_U^f) + \alpha(T_T^f - T_U^f)\left(\frac{s_U + 2s_T}{s_U s_T}\right) + \delta M \left(\frac{s_U + 2s_T}{s_U(s_U + s_T)}\right) \\
&\quad -\delta M \frac{2}{s_U} && (66) \\
&= \alpha(T_T^f - T_U^f)\left(1 - \frac{s_U + 2s_T}{s_U + s_T}\right) - M\frac{\delta}{s_U + s_T} && (67) \\
&= \alpha(T_T^f - T_U^f)\left(1 - \frac{s_U + 2s_T}{s_U + s_T}\right) - M\delta\frac{s_U + 2s_T - 2s_U - 2s_T}{s_U(s_U + s_T)} && (68) \\
&= \alpha(T_T^f - T_U^f)\left(\frac{s_T}{s_U + s_T}\right) - M\frac{\delta}{s_U + s_T}. && (69)
\end{aligned}$$

This is clearly negative when $T_T^f \geq T_U^f$. On the other hand, when $T_T^f < T_U^f$ it follows from the regularity assumption that $M > (T_U^f - T_T^f)s_T\frac{\alpha}{\delta}$ and hence (multiplying both sides by $\frac{\delta}{s_U+s_T}$) that $M\frac{\delta}{s_U+s_T} > \alpha(T_U^f - T_T^f)\frac{s_T}{s_U+s_T}$. Q.E.D.

Hence, since a positive route toll can only reduce $N_T$ it follows that it is impossible to improve social welfare with a positive uniform toll. [The case of a negative route tolls is considered in Appendix A.]

## 3.2 Using a Time-Varying Step Toll

Though it is not possible, in general, to influence commuters to make the optimal departure-time choices without a continuously time-varying toll, it is of interest to consider toll structures with somewhat simpler temporal toll strcutures. One such toll is the single-step toll.

As shown in Arnott et al. (1990b), the optimal step toll on each route is given by:

$$\tau_i = \frac{\delta N_i}{2s_i} \tag{70}$$

and this toll should be in place during the interval $[t_i^+, t_i^-]$. The resulting equilibrium departure rate function is given by:

$$\overline{r}_i(t) \begin{cases} s_i + \frac{\beta s_i}{\alpha-\beta} & , t \in [t_{iq}, t_i^+ - \tau_i/\alpha) \\ 0 & , t \in [t_i^+ - \tau_i/\alpha, t_i^+) \\ s_i + \frac{\gamma s_i}{\alpha+\gamma} & , t \in [t_i^+, \tilde{t}_i) \\ s_i - \frac{\gamma s_i}{\alpha+\gamma} & , t \in [\tilde{t}_i, t_{iq'}) \\ 2s_i\tau_i/(\alpha+\gamma) & , t = t_{iq'} \end{cases} \tag{71}$$

where the "critical times" are now given by:

$$t_{iq} = t^* - T_i^f - \frac{\gamma}{\beta+\gamma}\left(\frac{N_i}{s_i}\right) + \frac{(\gamma-\alpha)\tau_i}{(\beta+\gamma)(\alpha+\gamma)} \tag{72}$$

$$t_i^+ = t_{iq} + \frac{\tau_i}{\beta} + T_i^f \tag{73}$$

$$t_i^- = t_{iq} + \frac{N_i}{s_i} - \frac{2\tau_i}{\alpha+\gamma} + T_i^f \tag{74}$$

Note that $r_i(t_{iq'})$ is actually a bulk departure at an instant in time and not a departure rate.[1]

We now consider the case in which only one route can be tolled. Letting:

$$\begin{aligned} A &= \frac{3(\beta+\gamma)(\alpha+\gamma)}{2(\beta+\gamma)(\alpha+\gamma)} - \frac{\beta(\alpha-\gamma)}{2(\beta+\gamma)(\alpha+\gamma)} && (76) \\ &= \frac{3}{2} - \frac{\beta(\alpha-\gamma)}{2(\beta+\gamma)(\alpha+\gamma)} && (77) \end{aligned}$$

and

[1] As shown in Arnott et al. (1990b), independent step-tolls do not result in the optimal route splits. To achieve these splits, a uniform toll must be placed on one of the routes. As shown in Bernstein and El Sanhouri (1994), when $T_2^f > T_1^f$ the optimal route toll is given by:

$$\pi = \frac{\alpha(\beta+\gamma)(\alpha+\gamma)}{3(\beta+\gamma)(\alpha+\gamma) - \beta(\gamma-\alpha)}(T_2^f - T_1^f). \tag{75}$$

$$B = \frac{2(\beta+\gamma)(\alpha+\gamma)}{2(\beta+\gamma)(\alpha+\gamma)} - \frac{\beta(\alpha-\gamma)}{2(\beta+\gamma)(\alpha+\gamma)} \quad (78)$$

$$= 1 - \frac{\beta(\alpha-\gamma)}{2(\beta+\gamma)(\alpha+\gamma)} \quad (79)$$

it follows from (72) that the total private cost on route $T$, $\mathcal{C}_T : \Re_+ \rightarrow \Re_{++}$ is given by:

$$\mathcal{C}_T(N_T) = \alpha T_T^f N_T + \frac{\delta B}{s_T} N_T^2 \quad (80)$$

and it follows from (70) - (74) that the total social cost on route $T$, $\mathcal{S}_T : \Re_+ \rightarrow \Re_{++}$, is given by:

$$\mathcal{S}_T = \alpha T_T^f N_T + \delta \frac{A N_T^2}{2 s_T}. \quad (81)$$

Further, as before, it follows that the total private cost on route $U$, $\mathcal{C}_U : \Re_+ \rightarrow \Re_{++}$ is given by:

$$\mathcal{C}_U(N_U) = \alpha T_U^f N_U + \frac{\delta}{s_U} N_u^2 \quad (82)$$

Given these results we now proceed as before. In this case, regularity is defined as follows:

**Definition 3.2** *A two-route network is said to be* regular *iff:*

$$T_U^f > T_T^f \Rightarrow M > \frac{\alpha}{\delta}\frac{s_T}{B}(T_U^f - T_T^f) \quad (83)$$

*and*

$$T_T^f > T_U^f \Rightarrow M > \frac{\alpha}{\delta} s_U (T_T^f - T_U^f). \quad (84)$$

We will also need the following somewhat stronger condition:

**Definition 3.3** *A two-route network is said to be* strongly regular *iff:*

$$T_U^f > T_T^f \Rightarrow M > \frac{\alpha}{\delta}\left(\frac{8}{3}s_T + \frac{4}{3}s_U\right)(T_U^f - T_T^f). \quad (85)$$

Again, regularity ensures that both routes are used in equilibrium Strong regularity ensures that when the tolled route has a lower free-flow travel time, the untolled route is attractive to a fairly large number of users (i.e., it is a resonable alternative).

In order to determine the optimal route split we must minimize $\mathcal{T} = \mathcal{S}_T + \mathcal{C}_U$ subject to the constraint that $M = N_T + N_U$. Substituting for $N_U$ yields the following problem in one variable ($N_T$):

$$\min \mathcal{T} = \alpha T_T^f N_T + \delta \frac{AN_T^2}{2s_T} + \alpha T_U^f (M - N_T) + \delta \frac{(M - N_T)^2}{s_U}. \tag{86}$$

Differentiating and solving yields:

$$\begin{aligned}
\frac{\partial \mathcal{T}}{\partial N_T} = 0 \quad &\Rightarrow \quad \alpha T_T^f - \alpha T_U^f + \delta \frac{AN_T^*}{s_T} - \delta ta \frac{2M}{s_U} + 2\delta \frac{N_T^*}{s_U} = 0 &(87)\\
&\Rightarrow \quad N_T^* \delta \left( \frac{A}{s_T} + \frac{2}{s_U} \right) = \delta \frac{2M}{s_U} - \alpha (T_T^f - T_U^f) &(88)\\
&\Rightarrow \quad N_T^* \left( \frac{A}{s_T} + \frac{2}{s_U} \right) = \frac{2M}{s_U} + \frac{\alpha}{\delta} (T_U^f - T_T^f). &(89)
\end{aligned}$$

Hence:

$$N_T^* = \frac{\frac{2M}{s_U} + \frac{\alpha}{\delta}(T_U^f - T_T^f)}{\left( \frac{A}{s_T} + \frac{2}{s_U} \right)}. \tag{90}$$

And, since it follows from (77) and (79) that $A = B + \frac{1}{2}$, it also follows that:

$$N_T^* = \frac{M \frac{2\delta}{s_U} + \alpha (T_U^f - T_T^f)}{\delta \left( \frac{2}{s_U} + \frac{B+1/2}{s_T} \right)}. \tag{91}$$

On the other hand, the equilibrium route split can be determined by setting $C_T = C_U$ and solving for $\overline{N}_T$ as follows:

$$\begin{aligned}
\alpha T_T^f + \delta \frac{B\overline{N}_T}{s_T} = \alpha T_U^f + \delta \frac{\overline{N}_U}{s_U} \quad &\Rightarrow \quad \alpha T_T^f + \delta \frac{B\overline{N}_T}{s_T} = \alpha T_U^f + \delta \frac{M - \overline{N}_T}{s_U} &(92)\\
&\Rightarrow \quad \overline{N}_T \delta \left( \frac{B}{s_T} + \frac{1}{s_U} \right) = \alpha (T_U^f - T_T^f) + \delta \frac{M}{s_U}. &(93)
\end{aligned}$$

Hence:

$$\overline{N}_T = \frac{\alpha (T_U^f - T_T^f) + \delta \frac{M}{s_U}}{\delta \left( \frac{B}{s_T} + \frac{1}{s_U} \right)} \tag{94}$$

With this, it is possible to demonstrate the following:

**Lemma 3.2** *(i). On a regular network with the optimal step toll on route $T$, when $T_T^f \geq T_U^f$ the equilibrium number of users of route $T$ is less than the optimal number (i.e., $\overline{N}_T < N_T^*$).*

*(ii). On a strongly regular network with the optimal step toll on route $T$, the equilibrium number of users of route $T$ is always less than the optimal number (i.e., $\overline{N}_T < N_T^*$).*

**Proof.** Observe that:

$$\overline{N}_T < N_T^* \quad \Leftrightarrow \quad \frac{M\frac{\delta}{s_U} + \alpha(T_U^f - T_T^f)}{\delta\left(\frac{1}{s_U} + \frac{B}{s_T}\right)} < \frac{M\frac{2\delta}{s_U} + \alpha(T_U^f - T_T^f)}{\delta\left(\frac{2}{s_U} + \frac{B+1/2}{s_T}\right)} \tag{95}$$

$$\Leftrightarrow \quad M\frac{1}{s_U}\left[\frac{1}{\left(\frac{1}{s_U} + \frac{B}{s_T}\right)} - \frac{2}{\left(\frac{2}{s_U} + \frac{B+1/2}{s_T}\right)}\right] < \frac{\alpha}{\delta}(T_U^f - T_T^f)\left[\frac{1}{\left(\frac{2}{s_U} + \frac{B+1/2}{s_T}\right)} - \frac{1}{\left(\frac{1}{s_U} + \frac{B}{s_T}\right)}\right] \tag{96}$$

$$\Leftrightarrow \quad M\frac{1}{s_U}\left[\frac{1}{\frac{s_T + Bs_U}{s_U s_T}} - \frac{2}{\frac{2s_T + s_U(B+1/2)}{s_U s_T}}\right] < \frac{\alpha}{\delta}(T_U^f - T_T^f)\left[\frac{1}{\frac{2s_T + s_U(B+1/2)}{s_U s_T}} - \frac{1}{\frac{Bs_U + s_T}{s_u s_T}}\right] \tag{97}$$

$$\Leftrightarrow \quad M\frac{1}{s_U}\left[\frac{s_U s_T}{s_T + Bs_U} - \frac{2s_U s_T}{2s_T + s_U(B+1/2)}\right] < \frac{\alpha}{\delta}(T_U^f - T_T^f)\left[\frac{s_U s_T}{2s_T + s_U(B+1/2)} - \frac{s_U s_T}{s_T + Bs_U}\right] \tag{98}$$

$$\Leftrightarrow \quad M\frac{s_U s_T}{s_U}\left[\frac{1}{s_T + Bs_U} - \frac{2}{2s_T + s_U(B+1/2)}\right] < \frac{\alpha}{\delta}(T_U^f - T_T^f)s_u s_T\left[\frac{1}{2s_T + s_U(B+1/2)} - \frac{1}{s_T + Bs_U}\right] \tag{99}$$

$$\Leftrightarrow \quad M\left[\frac{[2s_T + s_U(B+1/2)] - 2(s_T + Bs_U)}{(s_T + Bs_U)[2s_T + s_U(B+1/2)]}\right] < \frac{\alpha}{\delta}s_U(T_U^f - T_T^f)\left[\frac{(s_T + Bs_U) - [2s_T + s_U(B+1/2)]}{(s_T + Bs_U)[2s_T + s_U(B+1/2)]}\right] \tag{100}$$

$$\Leftrightarrow \quad M < \frac{\alpha}{\delta}s_U(T_U^f - T_T^f)\left[\frac{(s_T + Bs_U) - [2s_T + s_U(B+1/2)]}{[2s_T + s_U(B+1/2)] - 2(s_T + Bs_U)}\right] \tag{101}$$

$$\Leftrightarrow \quad M > \frac{\alpha}{\delta}(T_T^f - T_U^f)s_U\left[\frac{(s_T + Bs_U) - [2s_T + s_U(B+1/2)]}{2(s_T + Bs_U) - [2s_T + s_U(B+1/2)]}\right] \tag{102}$$

$$\Leftrightarrow \quad M > \frac{\alpha}{\delta}(T_T^f - T_U^f)s_U\left[\frac{s_T + Bs_U - 2s_T - s_U B - 1/2 s_U}{2s_T + 2Bs_U - 2s_T - s_U B - 1/2 s_U}\right] \tag{103}$$

$$\Leftrightarrow \quad M > \frac{\alpha}{\delta}(T_T^f - T_U^f)s_U \left[\frac{-s_T - 1/2s_U}{s_U(B-1/2)}\right] \tag{104}$$

$$\Leftrightarrow \quad M > \frac{\alpha}{\delta}(T_U^f - T_T^f) \left[\frac{s_T + 1/2s_U}{(B-1/2)}\right] \tag{105}$$

$$\Leftrightarrow \quad M > \frac{\alpha}{\delta}(T_U^f - T_T^f) \left[\frac{2s_T + s_U}{(2B-1)}\right]. \tag{106}$$

Now, when $T_T^f \geq T_U^f$ this inequality is satisfied trivially since $\frac{7}{8} < B < 1$ implies that the right-hand side is non-positive. Hence (i) follows.

On the other hand, when $T_T^f < T_U^f$ we know from the assumption of strong regularity that $M > \frac{\alpha}{\delta}\left(\frac{8}{3}s_T + \frac{4}{3}s_U\right)(T_U^f - T_T^f)$. And, since:

$$\frac{7}{8} < B < 1 \Rightarrow \left[\frac{2s_T + s_U}{2B-1)}\right] < \frac{8}{3}s_T + \frac{4}{3}s_U \tag{107}$$

(ii) also follows. Q.E.D.

Thus, we immediately have the following result:

**Theorem 3.3** *On a strongly regular network with the optimal step-toll on route $T$, it is impossible to achieve the optimal route-split with a non-negative route toll (i.e., with a non-negative uniform toll).*

That is, when the network is strongly regular we cannot achieve both the (step-toll sub-) optimal departure-times on $T$ and the optimal route split between $T$ and $U$ if we leave route $U$ untolled.

# 4 Conclusion and Directions for Future Researc

It is well-understood that the benefits of facility-based congestion pricing are, in general, smaller when only a subset of the existing facilities can be tolled. For example, with route pricing, when more than one used route is left untolled, it may not be possible to achieve the optimal route splits. As another example, with departure-time pricing, when one route is left untolled it is impossible to achieve the optimal departure-time pattern on that route.

This paper has demonstrated that the inefficiencies introduced by untolled facilities may be worse than was originally suspected. In particular, we have demonstrated that:

> If the optimal one-route toll is in place (and hence the route splits are optimal) it is impossible to achieve the optimal mode split; and

If one route is left untolled and the optimal departure-time toll is in place on the other routes, it is impossible to achieve the optimal route split.

Of course, in and of itself, this is not an argument in favor of area pricing over facility-based pricing. Indeed, area pricing also results in inefficiencies since, in effect, the same toll is charged on different facilities [see, for example, Muller and Bernstein (1993)]. One is then left to ask how facility-based congestion pricing compares with area pricing. Hence, we will examine this issue in a future paper.

It is also worth considering whether there are alternative ways to introduce congestion pricing which would make it more attractive and possibly obviate the need for an untolled alternative. Though some such ideas have been discussed in the past [see, for example, Elliot (1986), Small, Winston and Evans (1989), Goodwin (1989) Jones (1991), Poole (1992), Small (1992), Bernstein (1993), and Muller and Bernstein (1993)], in a subsequent paper we will consider how the new technologies associated with Intelligent Transportation Systems (ITS) might be used to achieve the same goal. In particular, we will consider how congestion pricing might be introduced with a driver information system in order to increase its acceptability, and what this implies for the performance of the combined system.

# A The Optimal (Negative) Route Toll

As we saw above, the optimal non-negative route toll is zero. However, as discussed in Bernstein (1993), in some instances it may be possible to implement negative tolls. Hence, in this appendix we derive the optimal (negative) route toll (which is equivalent to the optimal route toll for route $U$).

The optimal route toll roll on route $U$, $\pi \in \Re_{++}$, is given by:

$$\pi = C_T(N_T^*) - C_U(N_U^*). \tag{108}$$

However, since at $N^* = (N_1^*, N_2^*)$ it must be the case that $C_T(N_T^*) = \frac{\partial \mathcal{S}_U(N_U^*)}{\partial N_U^*}$ it follows that:

$$\begin{aligned} \pi &= \alpha T_U^f + \delta \frac{N_U^*}{2s_U} - \alpha T_U^f - \delta \frac{N_U^*}{s_U} && (109)\\ &= \delta \frac{N_U^*}{2s_U} && (110)\\ &= \delta \left( \frac{M - N_T^*}{2s_U} \right) && (111) \end{aligned}$$

$$= \quad \delta\left[\frac{M-\left(\frac{2Ms_T}{s_U+2s_T}\right)}{2s_U}\right] \tag{112}$$

$$= \quad \delta\left[\frac{\left(\frac{Ms_U}{s_U+2s_T}\right)}{2s_U}\right] \tag{113}$$

$$= \quad \delta\left[\frac{Ms_U}{2s_U(s_U+2s_T)}\right] \tag{114}$$

$$= \quad \delta\left[\frac{M}{2(s_U+2s_T)}\right]. \tag{115}$$

One instance of this result is considered by Braid (1987). He assumes that $s_T = s_U = s$ and shows that the resulting route toll is given by $-\frac{\delta M}{6s}$.

# References


[1] Arnott, R. (1979) "Unpriced Transport Congestion", *Journal of Economic Theory*, Vol. 21, pp. 294-316.

[2] Arnott, R., A. de Palma, and R. Lindsey (1990a) "Economics of a Bottleneck", *Journal of Urban Economics*, Vol. 27, pp. 111-130.

[3] Arnott, R., A. de Palma, and R. Lindsey (1990b) "Departure Time and Route Choice for the Morning Commute", *Transportation Research*, Vol. 24B, pp. 209-228.

[4] Bernstein, D. and El Sanhouri (1994) "A Note on 'Departure Time and Route Choice for the Morning Commute' ", *Transportation Research*, forthcoming.

[5] Bernstein, D. and J. Muller (1994) "Understanding the Competing Short-Run Objectives of Peak Period Road Pricing", *Transportation Research Record*, forthcoming.

[6] Bernstein, D. (1993) "Congestion Pricing with Tolls and Subsidies", *Proceedings of the Pacific Rim Transportation Technologies Conference*, Vol. II, pp. 145-151.

[7] Braid, R.M. (1987) "Peak-Load Pricing of a Transport Facility with an Unpriced Substitute", manuscript.

[8] Braid, R.M. (1989) "Uniform versus Peak-Load Pricing of a Bottleneck with Elastic Demand", *Journal of Urban Economics*, Vol. 26, pp. 320-327.

[9] d'Ouville, E.L. and J.F. McDonald (1990) "Optimal Road Capacity with a Suboptimal Congestion Toll", *Journal of Urban Eoconomics*, Vol. 28, pp. 34-49.

[10] Elliot, W. (1986) "Fumbling Toward the Edge of History: California's Quest for a Road-Pricing Experiment", *Transportation Research*, Vol. 20A, pp. 151-156.

[11] El Sanhouri, I. (1994) "Evaluating the Joint Implementation of Congestion Pricing and Driver Information Systems", Ph.D. Dissertation, Massachusetts Institute of Technology.

[12] Goodwin, P.B. (1989) "The 'Rule of Three': A Possible Solution to the Political Problem of Competing Objectives for Road Pricing", *Traffic Engineering and Control*, Vol. 29, 1989, pp. 495-497.

[13] Hau, T. (1992) "Congestion Charging Mechanisms: An Evaluation of Current Practice", *World Bank Policy Research Working Paper Series*.

[14] Jones, P. (1991) "Gaining Public Support for Road Pricing Through a Package Approach", *Traffic Engineering and Control*, Vol. 32, pp. 194-196.

[15] Marchand, M. (1968) "A Note on Optimal Tolls in an Imperfect Environment", *Econometrica*, Vol. 36, pp. 575-581.

[16] Muller, J. and D. Bernstein (1993) "User-Neutral Congestion Pricing Schemes", submitted to *Transportation*.

[17] Poole, R.W. (1992) "Introducing Congestion Pricing on a New Toll Road", *Transportation*, Vol. 19, pp. 383-396.

[18] Small, K.A., C. Winston, and C.A. Evans (1989) *Road Work: A New Highway Pricing and Investment Policy*, The Brookings Institution, Washington, D.C.

[19] Small, K.A. (1992) "Using the Revenues from Congestion Pricing", *Transportation*, Vol. 19, pp. 359-381.

[20] Sullivan, A.M. (1983) "Second-Best Policies for Congestion Externalities", *Journal of Urban Economics*, Vol. 14, pp. 105-123.

[21] Vickrey, W. (1969). Congestion theory and transport investment. *American Economic Review*. **56**, 251-260.

[22] Wilson, J.D. (1983) "Optimal Road Capacity in the Presence of Unpriced Congestion", *Journal of Urban Economics*, Vol. 13, pp. 337-357.