Appendix D

A Brief Introduction to Waves

In order to understand the perception of auditory and visual content it is necessary to understand the physics of sound and light. In order to understand sound and light it is necessary to understand waves.

D.1 Mechanical Waves

A mechanical *pulse* is a single disturbance that moves through a a sequence of interacting particles (called a *medium*).[1] A mechanical *periodic wave* is a periodic disturbance that moves through a medium, transporting energy as it moves.

It is important to remember that the individual particles do not move very far. Each particle oscillates around its equilibrium position; its average position does not change. As a particle interacts with its neighbors it transfers some of its energy to them, causing them to oscillate. As this process continues, the energy is transported throug the medium.

Your first meaningful exposure to this phenomenon was probably water waves. Hence, much of your intution about waves comes from your experience with water waves. However, we are going to start instead with waves in a spring.

We view a spring as a medium consisting of individual coils. They are interesting because there are two ways to generate waves in a spring, both of which become important in the dicussions that follow.

First suppose that the wave is generated my moving the left end of the spring "back and forth" in the horizontal direction. This creates a series of *compressions* (areas in which the particles are closer than in equilibrium) and *rarefactions* areas in which the particles are farther apart than in equilibrium). If we take a "snapshot" of a spring being moved "back and forth", it might look like the illustration in Figure D.1-1 on the next page.

[1]As discussed in Section 1.2 on page 8, the word "medium" is used in a variety of different ways in different disciplines. We only use it to mean a sequence of interacting particles when discussing waves. Elsewhere, we use it in the sense of Definition 1.3 on page 10.

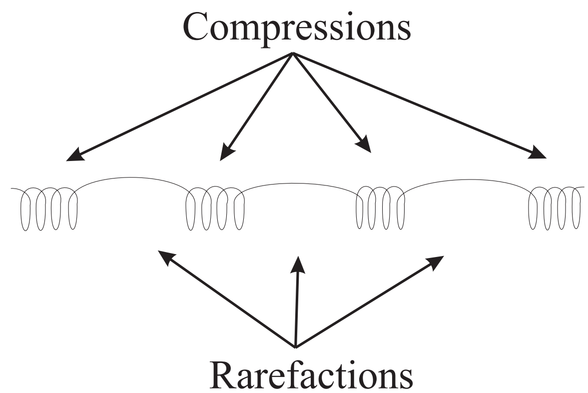

Figure D.1-1: A Longitudinal Wave

Moving the spring “back and forth” in this way generates a *longitudinal* wave; the particles (in this case, the coils) move parallel to the direction in which the energy is transferred. In other words, the particles move parallel to the direction in which the wave moves (i.e. both the particles and the wave move “back and forth”).

Now suppose that the wave is generated by moving the left end of the spring “up and down” in the vertical direction. This creates a series of *peaks* (areas in which the particles are “higher” than in equilibrium) and *troughs* areas in which the particles are “lower” than in equilibrium). If we take a “snapshot” of a spring being moved “up and down”, it might look like the illustration in Figure D.1-2 on the facing page.

Moving the spring “back and forth” in this way generates a *transverse* wave; the particles move perpenducular to the direction in which the energy is transferred. In other words, the particles move perpendicular to the direction in which the wave moves (i.e. the particles move “up and down” but the wave move “back and forth”).

D.2 Characterizing Waves in the Position Domain

Though the two kinds of waves, longitudinal and transverse, are physically very different, we can abstract away from those differences fairly easily. We do so by introducing the notion of the *amplitude* of a wave.

For a longitudinal wave in our spring, the amplitude is related to the number of coils per unit length. However, to “center” the amplitude, we subtract off the number of coils per unit length when the spring is in equilibrium (i.e. resting). This is illustrated in the Figure D.2-1 on the next page in which the spring is shown while being moved “back and forth” and while at rest.

The dotted rectangle in Figure D.2-1 on the facing page indicates the fixed unit of length used to measure the amplitude, and the vertical line indicates the position at which the amplitude is being measured. At the indicated position, there are two coils per unit length when the spring is at rest (i.e. in the bottom spring) and there are four

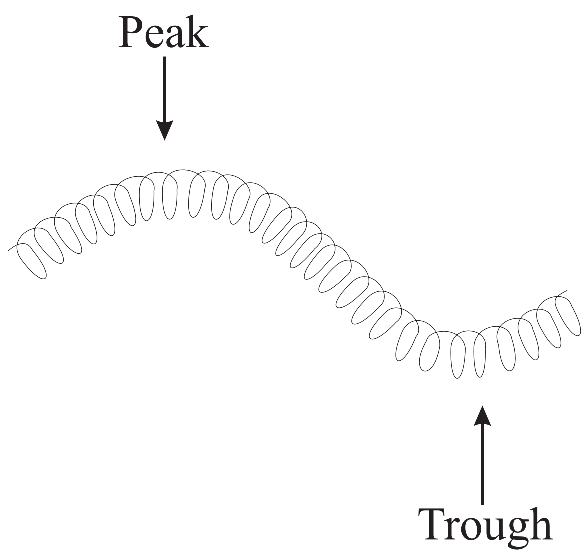

Figure D.1-2: A Transverse Wave

coils per unit length in the wave (i.e. in the top spring). This means that the amplitude at this position is two. The rectangle and line are then moved slightly and the process is repeated, yielding the graph in Figure D.2-2 on the next page.

For a transverse wave, the amplitude is related to the height of the coils. However, to "center" the amplitude, we subtract off the height of the coils when the spring is in equilibrium. This is illustrated in Figure D.2-3 on page 401 in which the spring is shown while being moved "up and down" and while at rest.

The vertical line in Figure D.2-3 on page 401 indicates the position at which the amplitude is being measured. (When the coil in the spring at rest is below the coil in the wave the amplitude is positive, in the opposite situation the amplitude is negative.) The line is then moved slightly and the process is repeated, yielding the graph in

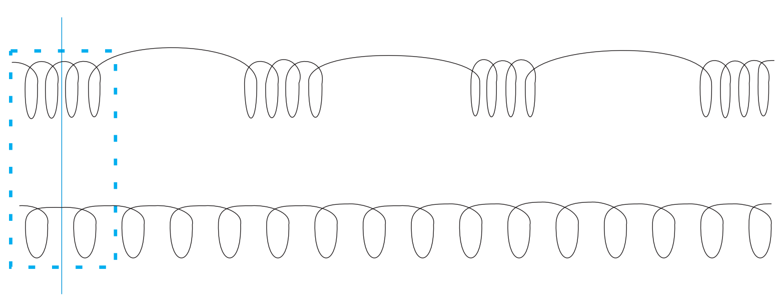

Figure D.2-1: A Spring: Moving and at Rest

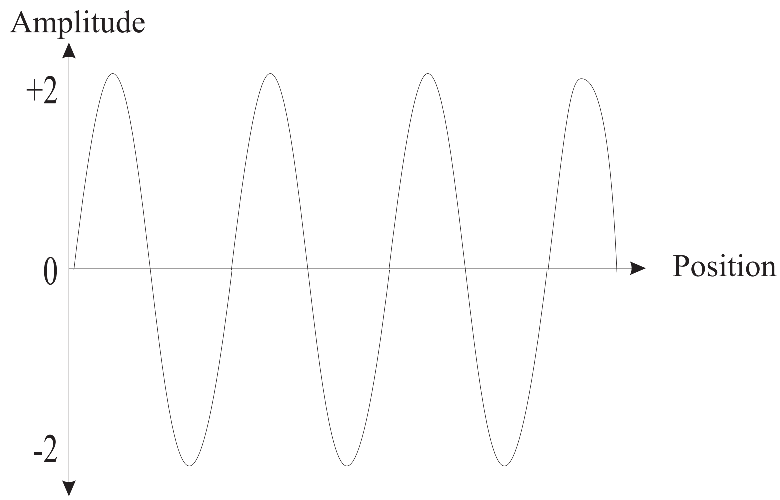

Figure D.2-2: A Graph of Amplitude versus Position for a Longitudinal Wave

Figure D.2-4 on page 402.

The *wavelength* of a periodic wave can be though of as the distance one has to travel along the wave until it "repeats". The wavelength is usually measured in meters (or parts thereof). and denoted by λ.

When two waves meet while traveleing through the same medium they are said to *interfere* with each other. The *principle of superposition* says that when two waves interfere, the resulting displacement of the medium at any location is the algebraic sum of the displacements of the individual waves at that same location. This leads is to distinguish between *constructive* interference (in which the displacements are both positive) and *destructive* intereference (in which one displacement is positive when the other is negative).

D.3 The Time Domain

The graphs discussed thus far involve the amplitude of the wave versus the position along the wave. That is, we picked a point in time and measured the amplitude of the wave at each position along the wave. When we use this approach we are considering the wave in the *position domain*.

Alternatively, since a wave varies periodically in time as well as space, we could have picked a particular position along the wave and measured the amplitude at that position over time. This approach leads to a graph of the amplitude versus time as shown in Figure D.3-1 on page 403. When we use this approach we are condiering the wave in the *time domain*.

In the time domain, a *cycle* is a portion of a wave from rest to crest to rest to trough to rest, and the *period* is the time required for a cycle (measured in cycles per second).

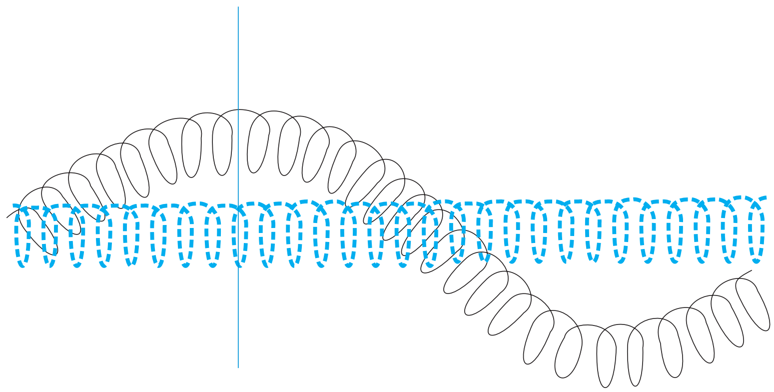

Figure D.2-3: A Spring: Moving and at Rest

The period can also be thought of as the time one has to wait at a position in space for the pattern to repeat.

The *frequency*, denoted by f, is the reciprocal of the period and, hence, is measured in cycles per second (i.e. *hertz*). The *speed* of a wave, denoted by v, is the product of its wavelength and frequency.[2] That is:

$$v = \lambda f \tag{D.1}$$

The speed (or velocity) of an object normally refers to its change in position over time. For waves, we have to choose a particular point on the wave. The easiest way to do this is to use a particular crest.

D.4 The Frequency Domain

So far we have considered the position domain and the time domain. While these approaches are often both useful and convenient, there are times when they are somewhat awkward to use. An alternative approach is to consider the *frequency domain*.

Figure D.4-1 on page 403 illustrates a 400Hz periodic wave in both the time domain (at the top) and the frequency domain (at the bottom). This particular example is called a *line spectrum* because the wave is strictly periodic. Quasiperiodic waves have *harmonic spectra* and aperiodic waves have *continuous spectra*.

Converting from the time domain to the frequency domain often involves the use of the Fourier transform. This approach is named after the mathematician who discovered that:

- All periodic waves may be expressed as the sum of a series of sinusoidal waves;

[2]Since we know the speed of light, we can easily calculate the wavelength of an electromagnetic wave from its frequency, and vice versa.

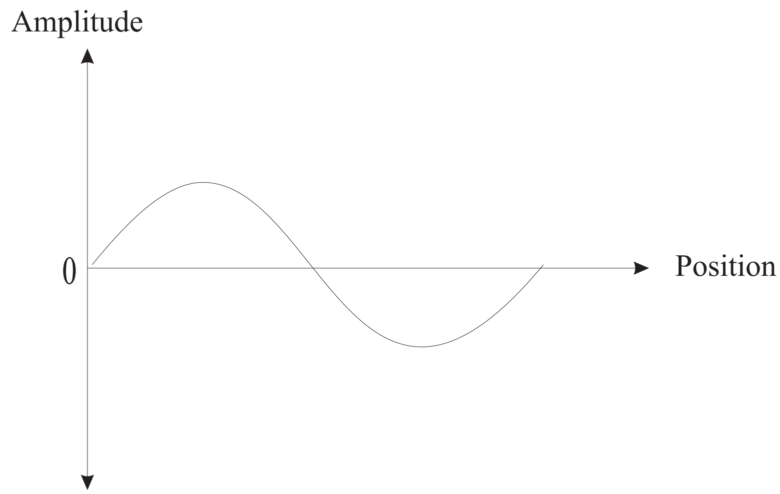

Figure D.2-4: A Graph of Amplitude verses Position for a Transverse Wave

- These waves are all integer multiples (called *harmonics*) of the fundamental frequency; and
- Each harmonic has its own amplitude and phase.

Fourier analysis is used to determine the component frequencies of a complicated wave. The way in which this is accomplished is beyond the scope of this book. However, the fact that a complicated wave can be represented as a sum of sinusoidal waves is relevant in several settings.

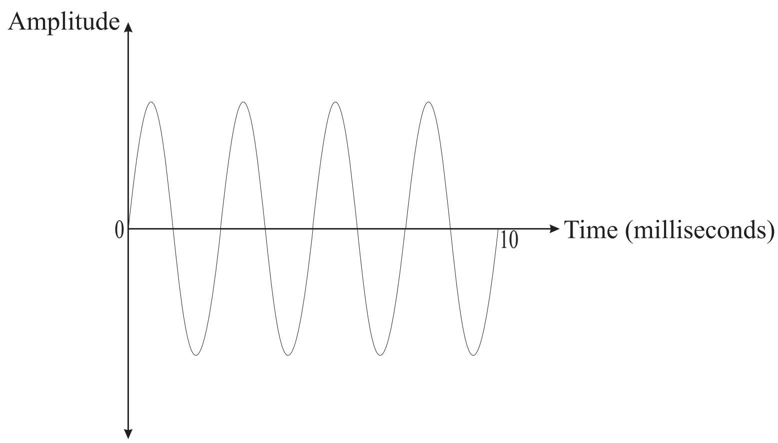

Figure D.3-1: A Graph of Amplitude versus Time

A 400Hz Periodic Wave

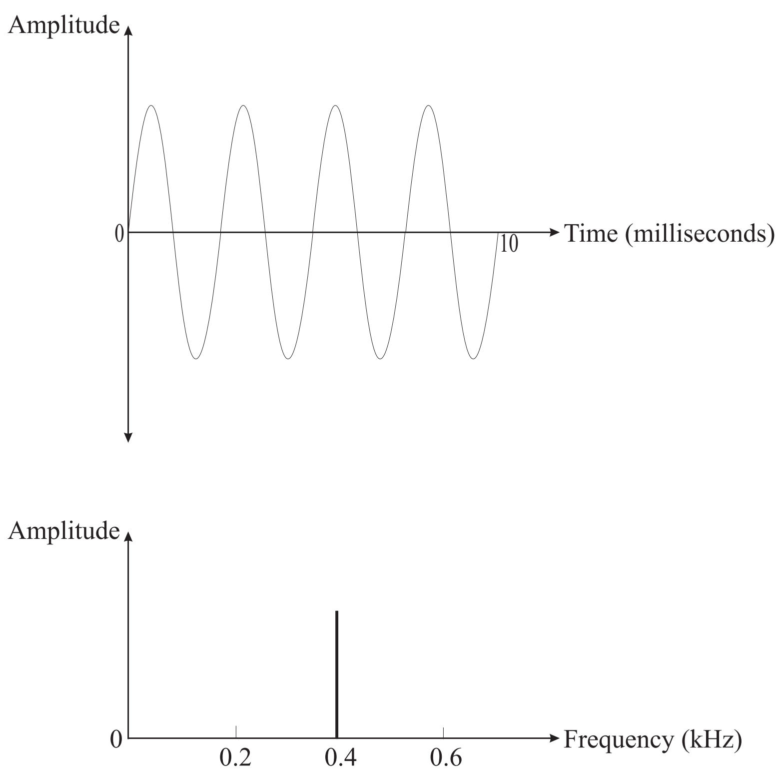

Figure D.4-1: A Wave in the Time and Frequency DOmains

Bibliography

Serway, R. A., Faughn, J. S., Vuille, C., and Bennet, C. A. (2005). *College Physics*. Brooks Cole, Boston, MA.