Study Guide for the Final Exam
CS-227, Spring 2009

NOTE that the material presented here is intended to assist you in your review and study of the course material. Overall, you are responsible for:

- **All** of the material in the **assigned readings,** whether or not it was covered in class.
- **All** of the material in the **assigned homework problems.**
- **All** of the material that was **covered in class,** whether or not it was also included in the assigned readings.

NOTE also that the Final Exam is **Cumulative.**

Material Covered after the Cutoff Point for the Mid-Term:

Elementary Number Theory

1. Divisibility (Rosen's Section 3.4)
    a. Factor: one number that divides another
    b. Multiple
    c. The Division Algorithm: **Dividend, Divisor, Quotient,** and **Remainder**
    d. Modular Arithmetic
    e. Congruence
    f. Pseudorandom Numbers
    g. Elementary Cryptography
        i. The Shift Cipher (*example*: The Caesar Cipher)
        ii. The Affine Cipher

2. Prime Numbers, and the Greatest Common Divisor (Rosen's Section 3.5)
    a. Two types of integers:
        i. Prime Numbers
        ii. Composite Numbers
    b. Fundamental Theorem of Arithmetic
    c. Searching for prime factors of a number s via brute force (trying all prime divisors up to the square root of the number)
    d. Greatest Common Divisor
    e. Least Common Multiple
    f. Pairwise Relative Primality

3. Representation of Integers (Rosen's Section 3.6
    a. Base Expansion of Integers

   b. Decimal Expansion
   c. Binary Expansion)
   d. Octal Expansion
   e. Hexadecimal Expansion
   f. Operations on Integers
      i. Addition
      ii. Multipliction
      iii. Integer Division
      iv. Modular Division
      v. Modular Exponentiation
   g. The Euclidian Algorithm

4. Applications of Number Theory (Rosen's Section 3.7
   a. The Extended Euclidian Algorithm
   b. Linear CongruencesThe Chinese Remainder Theorem
   c. Computer Arithmetic with Large Integers

)

5. Relations (Rosen's Section 8.1)
   a. Binary Relations
   b. Functions as Relations
   c. Relations on a Set
   d. Properties of Relations
      i. Reflexive Relations
      ii. Symmetric Relations
      iii. Antisymmetric Relations
      iv. Asymmetric Relations
      v. Transitive Relations
   e. Combinations of Relations
      i. Union
      ii. Intersection
      iii. Exclusive Or
      iv. DifferenceComposition of Relations

6. Relations among Elements from More Than TWO Sets (Rosen's Section 8.2)
   a. Relations of Degree Greater Than Two
   b. The Relational Data Model
      i. Records composed of *n*-tuples
      ii. Fields
      iii. Tables, Columns, Attributes
      iv. Primary Key
      v. Composite Key
      vi. Extension and Intension of a relation
   c. Operations on Relations of degree greater than two:

1. Selection
2. Projection
3. Join

d. Structured Query Language (SQL)

7. Representation of a Relation (Rosen's Section 8.3)
   a. Matrix
   b. Digraph

8. Basics of Counting (Rosen's Section 5.1)
   a. Product Rule
   b. Sum Rule
   c. Inclusion-Exclusion

9. The Pigeonhole Principle (Rosen's Section 5.2)

10. Permutations and Combinations (Rosen's Section 5.3)
    a. Permutations
    b. Combinations

11. Binomial Theorem (Rosen's Section 5.4)

12. Representation of Numbers, and the Performance of Arithmetic in the Digital Computer (my tutorial on the subject)
    a. Binary Representation of Integers
       i. "Unsigned" (*i.e.*, Non-Explicitly-Signed) Numbers
       ii. Ones'-Complement
       iii. Two's-Complement
       iv. Signed-Magnitude
       v. Excess-N
    b. Notation for Arithmetic Operations:
       i. Addition: Augend, Addend, Sum
       ii. Subtraction: Minuend, Subtrahend, Difference
       iii. Multiplication: Multiplicand, Multiplier, Product
       iv. Division: Dividend, Divisor, Quotient, Remainder
    c. Addition of Binary Integers:
       i. Carry Bits
       ii. Content of Sum Register
       iii. Detection of Overflow, if Present
       iv. Disposition of the Carry-out Bit from Addition in the Leftmost Bit Position

13. IEEE 754 Floating-Point Number Representation

Material Covered BEFORE the Cutoff Point for the Mid-Term:

Logic and Proofs

14. Propositional Logic (Rosen's Section 1.1)
    a. Operations:
        i. Conjunction (logical AND)
        ii. Disjunction (logical OR – *i.e.,* inclusive OR)
        iii. Exclusive OR
        iv. Conditional Statements (Implication)
        v. Biconditional
    b. Alternative notations
    c. Truth Tables (Tables 1 through 6 and Table 8 in Section 1.1)
        NOTE that I require that your Truth Tables be written with zeroes and ones in place of F's and T's, and that the rows be in lexicographic order.
    d. Logic Puzzles

15. Propositional Equivalence (Rosen's Section 1.2): Proof of equivalence via Truth Table

16. Predicates, Propositional Functions, and Quantifiers (Rosen's Section 1.3)
    a. Universal Quantifier
    b. Existential Quantifier
    c. Negation of a Quantified Expression

17. Validity of Arguments, Rules of Inference, and Fallacies (Rosen's Section 1.5)
    a. Table 1 on page 66

18. Proofs (Rosen's Section 1.6)
    a. Terminology:
        i. Axiom or Postulate
        ii. Proposition
        iii. Theorem
        iv. Lemma
        v. Corollary
        vi. Conjecture
        vii. Converse
        viii. Inverse
        ix. Contrapositive

b. Direct Proof
c. Indirect Proof
d. Proof by Contraposition
e. Proof by Contradiction

Basic Structures: Sets, Functions, Sequences, and Sums

19. Sets (Rosen's Section 2.1): DEFINITIONS and NOTATION
    a. Set
    b. Subset
    c. Proper Subset
    d. Power Set

20. Set Operations (Rosen's Section 2.2)
    a. Union
    b. Intersection
    c. Cartesian Product
    d. Venn Diagram
    e. Set Identities (Table 1 on page 124)
    f. Principle of Inclusion-Exclusion

21. Functions (Rosen's Section 2.3)
    a. Definition of FUNCTION
    b. One-to-One (Injective)
    c. Onto (Surjective)
    d. One-to-One Correspondence (Bijective)
    e. Increasing and Strictly Increasing
    f. Decreasing and Strictly Decreasing
    g. Invertible Function
    h. Composition of Functions
    i. Graph of a Function
    j. Special Functions:
        i. Floor Function
        ii. Ceiling Function
        iii. Factorial

22. Sequences and Summations (Rosen's Section 2.4)
    a. Sequence: DEFINITION
    b. Geometric Progression
    c. Arithmetic Progression
    d. Special Integer Sequences
    e. Summations
    f. Cardinality
    g. Countability and Uncountablility of an infinite set