CS-227, Discrete Structures I
Spring 2006 Semester

Summary of Course Coverage

1) Propositional Calculus
    a) Negation (logical NOT)
    b) Conjunction (logical AND)
    c) Disjunction (logical ***in***clusive-OR)
    d) Inequalities
    e) Propositional Form
    f) Truth Table
    g) Logical Equivalence
    h) Double Negative
    i) Negation of AND and of OR: DeMorgan's Laws and their application
    j) Inequalities and DeMorgan's Laws
    k) Tautology
    l) Contradiction
    m) Logical Equivalences (**NOTE:** This is summarized in Theorem 1.1.1 on p. 14.)
    n) Simplification of Logical Statements
    o) Conditional Statement: Hypothesis and Conclusion
    p) Vacuous Truth (Truth by Default)
    q) Order of Precedence of Logical Operators
    r) Negation of a Conditional Statement
    s) Forms Related to a Conditional Statement
        i) Converse
        ii) Inverse
        iii) Contrapositive
    t) Biconditional (Only-If)
    u) Necessary Conditions
    v) Sufficient Conditions

(continued)

2) Forms of Arguments: *either* valid *or* invalid
   a) Premises/Hypotheses/Assumptions, and Conclusions
   b) Valid Forms of Argument (**NOTE:** These are summarized in Table 1.3.1 on p. 40.)
      i) ***Modus Ponens*** (Method of Affirming)
      ii) ***Modus Tollens*** (Method of Denying)
      iii) Generalization
      iv) Specialization
      v) Transitivity
      vi) Division into Cases
   c) Fallacies or Invalid Forms of Argument
      i) Using Ambiguous Premises
      ii) Begging the Question
      iii) Jumping to a Conclusion
      iv) Converse Error
      v) Inverse Error

3) Digital Logic Circuits
   a) **Buffer Gate:** Logical Concept, Standard for Drawing, Standard for Labeling
   b) **NOT Gate:** Logical Concept, Standard for Drawing, Standard for Labeling
   c) **AND Gate:** Logical Concept, Standard for Drawing, Standard for Labeling
   d) **[Inclusive-] OR GATE:** Logical Concept, Standard for Drawing, Standard for Labeling
   e) Relationship between Truth Table, Boolean Expression, and Digital Logic Circuit
      i) [NOT-] AND-OR ("Sum of Products") Circuit
      ii) [NOT-] OR-AND ("Product of Sums") Circuit
   f) **NAND Gate** and its logical equivalent, the **Invert-OR Gate:** Logical Concept, Standard for Drawing, Standard for Labeling
   g) **NOR Gate** and its logical equivalent, the **Invert-AND Gate:** Logical Concept, Standard for Drawing, Standard for Labeling
   h) Straightforward Simplification of Boolean Logic Circuits
      i) **NAND-NAND** equivalent to each and every "Sum of Products" Circuit
      ii) **NOR-NOR** equivalent to each and every "Product of Sums" Ciircuit

4) Representation of Numbers
   a) Positional Representational Schemes for Number Representation
      i) Radix ***R***
      ii) Numerals: $0 \rightarrow (\boldsymbol{R} — 1)$
      iii) Radix Point to separate integer part of the number from fractional part
   b) Decimal Number Representation (generalized Radix Point becomes the Decimal Point)
   c) Binary Number Representation (generalized Radix Point becomes the Binary Point): Non-Negative Numbers and Numbers of Mixed Sign

- i) "Unsigned" (i.e., Non-Explicitly-Signed) Numbers
- ii) Signed-Magnitude
- iii) Ones'-Complement
- iv) Two's-Complement

d) Inter-Conversion of Binary, Octal, and Hexadecimal Number Representations (NOTE that generalized Radix Point becomes Octal Point or Hexadecimal Point for those representations)

e) Use of Hexadecimal Notation to Describe the Contents of a Register or of a Memory Location

f) Binary Addition
- i) The Half-Adder
- ii) The Full-Adder

g) The Four Processor-Status Bits, their Meaning and Significance:
- i) **C** bit
- ii) **Z** bit
- iii) **N** bit
- iv) **V** bit

h) Specification of the Content of Each Status Bit for each of the Schemes of Binary Number Representation
- i) **C** bit: What, if anything, does it signify?
- ii) **Z** bit: How many **Z**eroes?
- iii) **N** bit
- iv) **V** bit: Recognizing the occurrence of o**V**erflow.

5) Predicate Calculus

a) Definition of Predicate

b) Domain of a Predicate Variable
- i) The Set of [Mathematically] Real Numbers: $\boldsymbol{R}$
- ii) The Set of Integers: $\boldsymbol{Z}$, $\boldsymbol{Z}^{+}$, $\boldsymbol{Z}^{-}$, $\boldsymbol{Z}^{nonneg}$
- iii) The Set of Rational Numbers (*i.e.*, numbers expressible as ***Q***uotients of two integers): $\boldsymbol{Q}$

c) The Truth Set of a Predicate

d) The Universal Qualifier: $\forall$

e) The Existential Qualifier: $\exists$

f) Equivalence of Truth Sets: $\Rightarrow$ and $\Leftrightarrow$

g) Negation of a Universal Statement

h) Negation of an Existential Statement

i) Relationships among: $\forall$, $\exists$, $\wedge$, and $\vee$

j) Negation of a Universal ***Conditional*** Statement

k) Vacuous Truth

l) A Predicate and its Inverse, Converse, and Contrapositive

m) Necessary Conditions, Sufficient Conditions, and Only-If in Quantified Statements

6) Elementary Number Theory
    a) Even Integers: Formal Definition
    b) Odd Integers: Formal Definition
    c) Prime Numbers: Formal Definition
    d) Composite Numbers: Formal Definition
    e) Constructive Proof of Existence: Find an Example
    f) Disproof of a Universal Statement by means of a Counterexample
    g) Rules for Writing Formal Proofs: described in detail on page 134
    h) Common Mistakes made in the course of attempting a proof:
        i) Generalization from an example
        ii) Absent-Minded Use of a Single Variable in Two or More Definitions
        iii) Jumping to a Premature Conclusion
        iv) Begging the Question
    i) Rationality of the Sum of Two Rational Numbers
    j) Rationality of the Difference between Two Rational Numbers
    k) Rationality of the Product of Two Rational Numbers
    l) Retention of Integer Property under Addition, Subtraction, and Multiplication (but ***not*** under division)
    m) Divisibility
    n) Fundamental Theorem of Arithmetic (a.k.a. the Unique Factorization Theorem)

Mid-Term coverage ended here.

    o) Quotient-Remainder Theorem
    p) Types of Proofs I: Direct Proof
    q) Types of Proofs II: Disproof *via* Counterexample
    r) Types of Proofs III: Division Into Cases
    s) Types of Proofs IV: Contradiction
    t) Types of Proofs V: Contraposition
    u) Types of Proofs VI: Simple Mathematical Induction
    v) Types of Proofs VII: Strong Mathematical Induction

    w) Binary Representation of an Integer
    x) Parity
    y) Opposite Parity of Consecutive Integers
    z) Floor Function
    aa) Ceiling Function
    bb) Non-Existence of a Greatest Integer
    cc) Mutual Exclusivity of the Properties Even and Odd
    dd) Mixed Addition of Rational and Irrational Numbers

   ee) Irrationality of Square Root of 2
   ff) Infinitude of the Set of Prime Numbers

7) Algorithms
   a) Finding the Greatest Common Divisor: the Euclidean Algorithm
   b) Conversion from Decimal Representation to Another Radix I: Integer Part
   c) Conversion from Decimal Representation to Another Radix II: Fractional Part
   d) Pre-Conditions for Loops
   e) Post-Conditions for Loops
   f) The Loop Invariant
   g) Inevitability of Exit from the Loop
   h) Correctness of a Loop to Compute a Product
   i) Correctness of the Division Algorithm
   j) Correctness of the Euclidian Algorithm

8) Sequences
   a) Primitive Specification of terms: Enumeration
   b) Symbolic Specification of Terms: Explicit Formula
   c) Summation Notation *vs.* Expanded Notation
   d) Index of Summation, Lower Limit, Upper Limit
   e) Computing a Summation
   f) Interchange between Expanded Form and Summation Notation
   g) Separating Off of a Final Term
   h) Adding On of a Final Term
   i) Product Notation
   j) Computing a Product
   k) Properties of Summations and of Products
   l) Change of Variables
   m) Factorial Notation
   n) Computing Factorials
   o) Sum of First ***n*** Consecutive [Positive/Non-Negative] Integers
   p) Sum of a Geometric Sequence
   q) A Property of Divisibility Proved by Mathematical Induction
   r) An Inequality Proved by Mathematical Induction
   s) The Well-Ordering Principle of Integers
   t) Divisibility of Every Integer by a Prime Number
   u) Quotient-Remainder Theorem

9) Set Theory
   a) Definition I: Set

b) Definition II: Element
c) Irrelevance of the Order in which the elements are listed
d) Irrelevance of Multiple Listings of a single element
e) Notation: {}
f) Specification of Set Contents *via* Enumeration
g) Specification of Set Contents *via* Property Definition
h) Subset: Definition
i) Proper Subset: Definition
j) Set Equality
k) Venn Diagrams
l) Set Operations I: Union $\cup$
m) Set Operations II: Intersection $\cap$
n) Set Operations III: Difference —
o) Set Operations IV: Complementation ^{c}
p) The Empty Set (a.k.a. Null Set): {} or $\varnothing$
q) Disjoint Sets: Mutually Disjoint, Pairwise Disjoint
r) Partition of a Set
s) Power Set
t) The Ordered Pair
u) Cartesian Product
v) Set Identities: Analogous to Logical Identities