**Graphical representations of relations**

**(CS 227 class notes prepared by Ramon A. Mata-Toledo)**

Relations can be represented using directed graphs (or diagraphs).

A directed graph (or diagraph) G = (V, E) consists of a nonempty set of Vertices V and a set of directed edges (or arcs) E. Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair of vertices (*u*, *v*) is said to start at vertex *u* and end at vertex *v*.

The edges of a directed graph are depicted as arrows starting at vertex *u* and ending at vertex *v*. ■

## Representing relations by diagraphs

When representing relations by means of a graph the set of vertices of the graph is formed by the union of first and second projections of the graph. That is,

$$V = \Pr_1 R \cup \Pr_2 R$$

We will associate a directed edge from vertex *a* to vertex *b* if and only if (*a*,*b*) is an element of the relation. In other words, the edges of the directed graph will start at vertex *a* and end at vertex *b* if the ordered pair (*a*, *b*) is an element of the relation. ■

## Example No. 1

Find the directed graph associated with the relation $R = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

In this case $\Pr_1 R = \{1, 2, 3\}$ and $\Pr_2 R = \{1, 2, 3\}$. Therefore, V = {1, 2, 3}

The edges of the graphs are:

$1 \to 1$ $1\to2$ $2\to1$ $2\to2$ $2\to3$ $3\to1$ $3\to2$ $3\to3$

The graph may look like this

## Representing relations by matrices

A relation R can also be represented by a zero-one *n* x *m* matrix. The number of rows, *n*, of the matrix is equal to the cardinality of $\Pr_1 R$. The number of columns, *m*, is equal to the cardinality of $\Pr_2 R$.

The entries of the matrix $M_R$ (matrix associated with the relation R) are defined as follows:

$$M_{Rij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

■

## Example No. 2

Find the matrix associated with the relation $R = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,3)\}$

The matrix associated with *R* is $M_R = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$

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**References**

This set of notes follows Discrete Mathematics and Its Applications by Rosen. In addition to the previous reference some material has been adopted from the following books:


1) Introduccion a la Teoria de Conjuntos by L. Oubiña. Editorial Universitaria de Buenos Aires,1965. (Spanish version only)

2) Elements of Mathematics:Theory of Sets by N. Bourbaki. Addison-Wesley Publishing Company, 1968. This book is a translation of Éléments de Mathématic, Théorie des ensembles, originally published in French by Hermann, Paris. No date available for the French translation.