Review Questions:

Digital Number Representation

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1. What is the significance of the use of the word "digit" to represent a numerical quantity?

2. Which radices are "natural" (i.e., they correspond to some aspect of human anatomy)? What is their anatomical tie-in?

3. What is the principal advantage of the (Zoroastrian) sexagesimal (base 60) number system?

4. How long ago did the concept of zero originate?

5.   (a) How many different numbers (or different anythings) can be represented by n bits?

   (b) How many different items are representable with 24 bits? 32? 48?

   (c) How could you rapidly determine how many different items are representable with any given number of bits that you might be asked about?

6. How should you go about converting a number *from decimal* representation to another radix?

7. Sample Problems: Convert each of the following decimal numbers to the radices indicated:

   (a)   2,346.1359 to radix 3, radix 9, and radix 16.
   (b)   7,924.9483 to radix 2, radix 5, and radix 11.
   (c)   1,781.1953 to radix 7, radix 8, and radix 13.

8. How should you go about converting *from other radices to decimal?*

9. Sample Problems: Convert each of the following numbers to decimal:

   (a) $111001010111.000101110111_2$

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    (b) 123.321_4
    (c) 123.4_7
    (d) 7123.4567_8
    (e) 123.4_9
    (f) 123.4_{11}
    (g) $E56.D78_{16}$

10. How should you interconvert among binary, octal, and hex?

11. Sample Problems: Each of the following numbers is written either in binary or in octal or in hexadecimal. Convert it to the other two radices.

    (a) 10101010101.0101010101_2
    (b) 10111011101.1101110111_2
    (c) 10101.0101_8
    (d) 11101.11011_8
    (e) 101.01_{16}
    (f) 1101.11_{16}
    (g) 12345.67124_8
    (h) 12345.67124_{16}
    (i) $DEAD.ACE_{16}$
    (j) $BAD.BEEF_{16}$

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Answers to Selected Questions:

Question 2:   Which radices are "natural" (i.e., they correspond to some aspect of human anatomy)? What is their anatomical tie-in?

Radix	Name of Scheme	Anatomical Basis
5	Quinary	Number of fingers on one hand
10	Decimal	Number of fingers on both hands
12	Duodecimal	Number of finger joints that can be pointed to from the thumb
20	Vigesimal	Total number of fingers and toes together
24		Number of finger joints that can be pointed to from both thumbs

Question 3:   What is the principal advantage of the (Zoroastrian) sexagesimal (base 60) number system?

Answer: The radix is divisible by 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30.

Question 4:   How   long   ago   did   the   concept   of   zero   originate?

Answer: About 2,300 years ago in primitive form, and about 1400 years ago in its full-blown form.

Question 5a: How many different numbers (or different anythings) can be represented by *n* bits?

Answer: 2^n

Examples:

n	*# of different numbers or other objects representable*	
0	1	
1	2	
2	4	
3	8	
4	16	
5	32	
6	64	
7	128	
8	256	
9	512	

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10	1,024	= 1k
11	2,048	= 2k
. . . .		
20	1,048,576	= 1M
. . . .		
30	1,073,741,824	= 1G
. . . .		
40	1,099,511,527,776	= 1T

Question 5b: How many different items are representable with 24 bits? 32? 48?

     Answer: 16M 4G   256T

Question 6: How should you go about converting a number *from decimal* representation to another radix?   *Answer:*

(a) Deal separately with the integer part and the fractional part.

(b) Integer part: Divide successively by the new radix. Each division produces a quotient and a remainder. The remainder from the nth division is the nth significant digit, **starting** from the **least** significant digit immediately to the left of the radix point. *N.B.:* If the remainder is greater than 9, be sure to convert it to a single digit appropriate to the new radix.

(c) Fractional part: Multiply successively by the new radix. Each successive multiplication produces a new product with an integer part and a fractional part. The integer part from the nth multiplication provides the nth significant fractional digit, **starting** from the radix point; the fractional part **only** is used for the next multiplication. *N.B.:* If the integer part of any multiplication is greater than 9, be sure to convert it to a single digit in the new radix.

(d) Stopping point for the fractional part:
      (i)   There is nothing more left to multiply.
      (ii)   A repeating group is recognized (be sure to indicate clearly in your answer both that there is a repeating group and what it is).
      (iii)   The required level of precision is reached. For **CS-350,** the required level of precision is three hex digits, four octal digits, or twelve bits; six digits in any other radix.

Question 7: Conversion of decimal numbers to indicated radices:

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Sub-question 7a: 2346.1359

Integer Part = 2346			
#	Division	Quotient	Remainder
1st	2346/3	782	0
2nd	782/3	260	2
3rd	260/3	86	2
4th	86/3	28	2
5th	28/3	9	1
6th	9/3	3	0
7th	3/3	1	0
8th	1/3	0	1

Fractional Part = .1359			
#	Multiplication	Product	Integer Part
1st	.1359 * 3	0.4077	0
2nd	.4077 * 3	1.2231	1
3rd	.2231 * 3	0.6693	0
4th	.6693 * 3	2.0079	2
5th	.0079 * 3	0.0237	0
6th	.0237 * 3	0.0711	0
7th	.0711 * 3	0.2133	0
8th	.2133 * 3	0.6399	0
9th	.6399 * 3	1.9197	1
10th	.9197 * 3	2.7591	2

Answer = $10012220.0102000012+_3$

Integer Part = 2346			
#	Division	Quotient	Remainder
1st	2346/9	260	6
2nd	260/9	28	8
3rd	28/9	3	1
4th	3/9	0	3

Fractional Part = .1359			
#	Multiplication	Product	Integer Part
1st	.1359 * 9	1.2231	1
2nd	.2231 * 9	2.0079	2
3rd	.0079 * 9	0.0711	0
4th	.0711 * 9	0.6399	0
5th	.6399 * 9	5.7591	5
6th	.7591 * 9	6.8319	6
7th	.8319 * 9	7.4871	7
8th	.4871 * 9	4.3839	4
9th	.3839 * 9	3.4551	3

Answer = $3186.120056743+_9$

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Integer Part = 2346			
#	Division	Quotient	Remainder
1st	2346/16	146	$10_{10} = A_{16}$
2nd	146/16	9	2
3rd	9/16	0	9

Fractional Part = .1359			
#	Multiplication	Product	Integer Part
1st	.1359 * 16	2.1744	2
2nd	.1744 * 16	2.7904	2
3rd	.7904 * 16	12.6464	$12_{10} = C_{16}$

Answer = $92A.22C+_{16}$

Sub-Question 7b: 7924.9483

Integer Part = 7924			
#	Division	Quotient	Remainder
1st	7924/2	3962	0
2nd	3962/2	1981	0
3rd	1981/2	990	1
4th	990/2	495	0
5th	495/2	247	1
6th	247/2	123	1
7th	123/2	61	1
8th	61/2	30	1
9th	30/2	15	0
10th	15/2	7	1
11th	7/2	3	1
12th	3/2	1	1
13th	½	0	1

Fractional Part = .9483			
#	Multiplication	Product	Integer Part
1st	.9483 * 2	1.8966	1
2nd	.8966 * 2	1.7932	1
3rd	.7932 * 2	1.5864	1
4th	.5864 * 2	1.1728	1
5th	.1728 * 2	0.3456	0
6th	.3456 * 2	0.6912	0
7th	.6912 * 2	1.3824	1
8th	.3824 * 2	0.7648	0
9th	.7648 * 2	1.5296	1
10th	.5296 * 2	1.0592	1
11th	.0592 * 2	0.1184	0
12th	.1184 * 2	0.2368	1

Answer = $1111011110100.111100101101+_{2}$

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Integer Part = 7924			
#	Division	Quotient	Remainder
1st	7924/5	1584	4
2nd	1584/5	316	4
3rd	316/5	63	1
4th	63/5	12	3
5th	12/5	2	2
6th	2/5	0	2

Fractional Part = .9483			
#	Multiplication	Product	Integer Part
1st	.9483 * 5	4.7415	4
2nd	.7415 * 5	3.7075	3
3rd	.7075 * 5	3.5375	3
4th	.5375 * 5	2.6875	2
5th	.6875 * 5	3.4375	3
6th	.4375 * 5	2.1875	2
7th	.1875 * 5	0.9375	0
8th	.9375 * 5	4.6875	4
9th	.6875 * 5	3.4375	3
10th	.4375 * 5	2.1875	2

Answer = $223144.43323320432+_5$

Integer Part = 7924			
#	Division	Quotient	Remainder
1st	7924/11	720	4
2nd	720/11	65	5
3rd	65/11	5	$10_{10}=A_{11}$
4th	5/11	0	5

Fractional Part = .9483			
#	Multiplication	Product	Integer Part
1st	.9483 * 11	10.4313	$10_{10}=A_{11}$
2nd	.4313 * 11	4.7443	4
3rd	.7443 * 11	8.1873	8
4th	.1873* 11	2.0603	2

Answer = $5A54.A4822_{11}$

Sub-Question 7c: 1781.1953

Integer Part = 1781			
#	Division	Quotient	Remainder
1st	1781/7	254	3
2nd	254/7	36	2
3rd	36/7	5	1
4th	5/7	0	5

Fractional Part = .1953			
#	Multiplication	Product	Integer Part
1st	.1953 * 7	1.3671	1
2nd	.3671 * 7	2.5697	2
3rd	.5697 * 7	3.9879	3
4th	.9879 * 7	6.9153	6
5th	.9153 * 7	6.4071	6
6th	.4071 * 7	2.8497	2
7th	.8497 * 7	5.9479	5
8th	.9479 * 7	6.6353	6

Answer = 5123.12366256_7

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Integer Part = 1781			
#	Division	Quotient	Remainder
1st	1781/8	222	5
2nd	222/8	27	6
3rd	27/8	3	3
4th	3/8	0	3

Fractional Part = .1953			
#	Multiplication	Product	Integer Part
1st	.1953 * 8	1.5624	1
2nd	.5624 * 8	4.4992	4
3rd	.4992 * 8	3.9936	3
4th	.9936 * 8	7.9488	7
5th	.9488 * 8	7.5904	7
6th	.5904 * 8	4.7232	4

Answer = $3365.143774+_8$

Integer Part = 1781			
#	Division	Quotient	Remainder
1st	1781/13	137	0
2nd	137/13	10	7
3rd	10/13	0	$10_{10} = A_{13}$

Fractional Part = .1953			
#	Multiplication	Product	Integer Part
1st	.1953 * 13	2.5389	2
2nd	.5389 * 13	7.0057	7
3rd	.0057 * 13	0.0741	0
4th	.0741 * 13	0.9633	0
5th	.9633 * 13	12.5229	$12_{10}=C_{13}$
6th	.5229 * 13	6.7977	6

Answer = $A70.2700C6+_{13}$

Question 9: Conversion of numbers from various radices to decimal:

    (a) $111001010111.000101110111_2$ = 3671.091552646875
    (b) 123.321_4 = 27.890625
    (c) 123.4_7 = 66.571428+
    (d) 7123.4567_8 = 3667.591554734375894.444. . .
    (e) 123.4_9 = 102.444. . .
    (f) 123.4_{11} = 3670.841796875000

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Question 10: How should you interconvert among binary, octal, and hex?

Partial Answer: **Always** outwards from the radix point, **and** add zeroes when needed to the most significant end of the integer part and to the least significant end of the fractional part. **NEVER** add a zero between the least significant integer digit and the radix point, or between the most significant fractional digit and the radix point.

Question 11: Interconversion among binary, octal, and hexadecimal:

(a) $2525.2524_8 = 555.554_{16}$
(b) $2735.6734_8 = 5DD.DDC_{16}$
(c) $001\ 000\ 001\ 000\ 001.000\ 001\ 000\ 001_2 = 1241.2412_{16}$
(d) $001\ 001\ 001\ 000\ 001.001\ 001\ 000\ 001\ 001_2 = 1241.2412_{16}$
(e) $0001\ 0000\ 0001.0000\ 0001_2 = 401.002_8$
(f) $0001\ 0001\ 0000\ 0001.0001\ 0001_2 = 10401.042_8$
(g) $001\ 010\ 011\ 100\ 101.110\ 111\ 001\ 010\ 100_2 = 14E5.DCA8_{16}$
(h) $0001\ 0010\ 0011\ 0100\ 0101.0110\ 0111\ 0001\ 0010\ 0100_2 = 221505.316111_8$
(i) $1101\ 1110\ 1010\ 1101.1010\ 1100\ 1110_2 = 157255.5316_8$
(j) $1011\ 1010\ 1101.1011\ 1110\ 1110\ 1111_2 = 5655.575674_8$

Revised 15 Sep 1998; corrected 19 Jan 2004