Review Questions:

Digital Number Representation

1. What is the significance of the use of the word "digit" to represent a numerical quantity?

2. Which radices are "natural" (i.e., they correspond to some aspect of human anatomy)? What is their anatomical tie-in?

3. What is the principal advantage of the (Zoroastrian) sexagesimal (base 60) number system?

4. How long ago did the concept of zero originate?

5. (a) How many different numbers (or different anythings) can be represented by n bits?

   (b) How many different items are representable with 24 bits? 32? 48?

   (c) How could you rapidly determine how many different items are representable with any given number of bits that you might be asked about?

6. How should you go about converting a number *from decimal* representation to another radix?

7. Sample Problems: Convert each of the following decimal numbers to the radices indicated:

   (a) 2,346.1359 to radix 3, radix 9, and radix 16.
   (b) 7,924.9483 to radix 2, radix 5, and radix 11.
   (c) 1,781.1953 to radix 7, radix 8, and radix 13.

8. How should you go about converting *from other radices to decimal?*

9. Sample Problems: Convert each of the following numbers to decimal:

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(a) $111001010111.000101110111_2$
(b) 123.321_4
(c) 123.4_7
(d) 7123.4567_8
(e) 123.4_9
(f) 123.4_{11}
(g) $E56.D78_{16}$

10. How should you interconvert among binary, octal, and hex?

11. Sample Problems: Each of the following numbers is written either in binary or in octal or in hexadecimal. Convert it to the other two radices.

(a) 10101010101.0101010101_2
(b) 10111011101.1101110111_2
(c) 10101.0101_8
(d) 11101.11011_8
(e) 101.01_{16}
(f) 1101.11_{16}
(g) 12345.67124_8
(h) 12345.67124_{16}
(i) $DEAD.ACE_{16}$
(j) $BAD.BEEF_{16}$

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Answers to Selected Questions:

Question 2:

Radix	Name of Scheme	Anatomical Basis
5	Quinary	Number of fingers on one hand
10	Decimal	Number of fingers on both hands
12	Duodecimal	Number of finger joints that can be pointed to from the thumb
20	Vigesimal	Total number of fingers and toes together
24		Number of finger joints that can be pointed to from both thumbs

Question 3: Radix is divisible by 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30.

Question 4: About 2,300 years ago in primitive form, and about 1400 years ago in its full-blown form.

Question 5a: 2^n

Examples:

n	***# of different numbers or other objects representable***	
0	1	
1	2	
2	4	
3	8	
4	16	
5	32	
6	64	
7	128	
8	256	
9	512	
10	1,024	= 1k
11	2,048	= 2k
. . . .		
20	1,048,576	= 1M
. . . .		
30	1,073,741,824	= 1G
. . . .		
40	1,099,511,527,776	= 1T

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Question 5b: 16M 4G 256T

Question 6:

(a) Deal separately with the integer part and the functional part.

(b) Integer part: Divide successively by the new radix. Each division produces a quotient and a remainder. The remainder from the n^{th} division is the n^{th} significant digit, **starting** from the **least** significant digit immediately to the left of the radix point. *N.B.:* If the remainder is greater than 9, be sure to convert it to a single digit appropriate to the new radix.

(c) Fractional part: Multiply successively by the new radix. Each successive multiplication produces a new product with an integer part and a fractional part. The integer part from the n^{th} multiplication provides the n^{th} significant fractional digit, **starting** from the radix point; the fractional part **only** is used for the next multiplication. *N.B.:* If the integer part of any multiplication is greater than 9, be sure to convert it to a single digit in the new radix.

(d) Stopping point for the fractional part:

- (i) There is nothing more left to multiply.
- (ii) A repeating group is recognized (be sure to indicate clearly in your answer both that there is a repeating group and what it is).
- (iii) The required level of precision is reached. For ***CS-350,*** the required level of precision is three hex digits, four octal digits, or twelve bits; six digits in any other radix.

Question 7a: 2346.1359

Integer Part = 2346			
#	Division	Quotient	Remainder
1^{st}	2346/3	782	0
2^{nd}	782/3	260	2
3^{rd}	260/3	86	2
4^{th}	86/3	28	2
5^{th}	28/3	9	1
6^{th}	9/3	3	0
7^{th}	3/3	1	0
8^{th}	1/3	0	1

Fractional Part = .1359			
#	Multiplication	Product	Integer Part
1^{st}	.1359 * 3	0.4077	0
2^{nd}	.4077 * 3	1.2231	1
3^{rd}	.2231 * 3	0.6693	0
4^{th}	.6693 * 3	2.0079	2
5^{th}	.0079 * 3	0.0237	0
6^{th}	.0237 * 3	0.0711	0
7^{th}	.0711 * 3	0.2133	0
8^{th}	.2133 * 3	0.6399	0
9^{th}	.6399 * 3	1.9197	1
10^{th}	.9197 * 3	2.7591	2

Answer = $10012220.0102000012+_3$

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Integer Part = 2346			
#	Division	Quotient	Remainder
1st	2346/9	260	6
2nd	260/9	28	8
3rd	28/9	3	1
4th	3/9	0	3

Fractional Part = .1359			
#	Multiplication	Product	Integer Part
1st	.1359 * 9	1.2231	1
2nd	.2231 * 9	2.0079	2
3rd	.0079 * 9	0.0711	0
4th	.0711 * 9	0.6399	0
5th	.6399 * 9	5.7591	5
6th	.7591 * 9	6.8319	6
7th	.8319 * 9	7.4871	7
8th	.4871 * 9	4.3839	4
9th	.3839 * 9	3.4551	3

Answer = $3186.120056743+_9$

3rd	9/16	0	9

Answer = $92A.22C+_{16}$

Integer Part = 2346			
#	Division	Quotient	Remainder
1st	2346/16	146	$10_{10} = A_{16}$
2nd	146/16	9	2

Fractional Part = .1359			
#	Multiplication	Product	Integer Part
1st	.1359 * 16	2.1744	2
2nd	.1744 * 16	2.7904	2
3rd	.7904 * 16	12.6464	$12_{10} = C_{16}$

Question 7b: 7924.9483

Integer Part = 7924			
#	Division	Quotient	Remainder
1st	7924/2	3962	0
2nd	3962/2	1981	0
3rd	1981/2	990	1
4th	990/2	495	0
5th	495/2	247	1
6th	247/2	123	1
7th	123/2	61	1
8th	61/2	30	1
9th	30/2	15	0
10th	15/2	7	1
11th	7/2	3	1
12th	3/2	1	1
13th	½	0	1

Fractional Part = .9483			
#	Multiplication	Product	Integer Part
1st	.9483 * 2	1.8966	1
2nd	.8966 * 2	1.7932	1
3rd	.7932 * 2	1.5864	1
4th	.5864 * 2	1.1728	1
5th	.1728 * 2	0.3456	0
6th	.3456 * 2	0.6912	0
7th	.6912 * 2	1.3824	1
8th	.3824 * 2	0.7648	0
9th	.7648 * 2	1.5296	1
10th	.5296 * 2	1.0592	1
11th	.0592 * 2	0.1184	0
12th	.1184 * 2	0.2368	1

Answer = $1111011110100.111100101101+_2$

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Integer Part = 7924			
#	Division	Quotient	Remainder
1st	7924/5	1584	4
2nd	1584/5	316	4
3rd	316/5	63	1
4th	63/5	12	3
5th	12/5	2	2
6th	2/5	0	2

Fractional Part = .9483			
#	Multiplication	Product	Integer Part
1st	.9483 * 5	4.7415	4
2nd	.7415 * 5	3.7075	3
3rd	.7075 * 5	3.5375	3
4th	.5375 * 5	2.6875	2
5th	.6875 * 5	3.4375	3
6th	.4375 * 5	2.1875	2
7th	.1875 * 5	0.9375	0
8th	.9375 * 5	4.6875	4
9th	.6875 * 5	3.4375	3
10th	.4375 * 5	2.1875	2

Answer = $223144.43323320432+_5$

Integer Part = 7924			
#	Division	Quotient	Remainder
1st	7924/11	720	4
2nd	720/11	65	5
3rd	65/11	5	$10_{10}=A_{11}$
4th	5/11	0	5

Fractional Part = .9483			
#	Multiplication	Product	Integer Part
1st	.9483 * 11	10.4313	$10_{10}=A_{11}$
2nd	.4313 * 11	4.7443	4
3rd	.7443 * 11	8.1873	8
4th	.1873* 11	2.0603	2

Answer = $5A54.A4822_{11}$

Question 7c: 1781.1953

Integer Part = 1781			
#	Division	Quotient	Remainder
1st	1781/7	254	3
2nd	254/7	36	2
3rd	36/7	5	1
4th	5/7	0	5

Fractional Part = .1953			
#	Multiplication	Product	Integer Part
1st	.1953 * 7	1.3671	1
2nd	.3671 * 7	2.5697	2
3rd	.5697 * 7	3.9879	3
4th	.9879 * 7	6.9153	6
5th	.9153 * 7	6.4071	6
6th	.4071 * 7	2.8497	2
7th	.8497 * 7	5.9479	5
8th	.9479 * 7	6.6353	6

Answer = 5123.12366256_7

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Integer Part = 1781			
#	Division	Quotient	Remainder
1st	1781/8	222	5
2nd	222/8	27	6
3rd	27/8	3	3
4th	3/8	0	3

Fractional Part = .1953			
#	Multiplication	Product	Integer Part
1st	.1953 * 8	1.5624	1
2nd	.5624 * 8	4.4992	4
3rd	.4992 * 8	3.9936	3
4th	.9936 * 8	7.9488	7
5th	.9488 * 8	7.5904	7
6th	.5904 * 8	4.7232	4

Answer = $3365.143774+_8$

Integer Part = 1781			
#	Division	Quotient	Remainder
1st	1781/13	137	0
2nd	137/13	10	7
3rd	10/13	0	$10_{10} = A_{13}$

Fractional Part = .1953			
#	Multiplication	Product	Integer Part
1st	.1953 * 13	2.5389	2
2nd	.5389 * 13	7.0057	7
3rd	.0057 * 13	0.0741	0
4th	.0741 * 13	0.9633	0
5th	.9633 * 13	12.5229	$12_{10}=C_{13}$
6th	.5229 * 13	6.7977	6

Answer = $A70.2700C6+_{13}$

Question 9:

(a) 3671.091552646875
(b) 27.890625
(c) 66.731428+
(d) 3667.591554734375894.444. . .
(e) 146.3636. . .
(f) 3670.841796875000

Partial Answer to 10: **Always** outwards from the radix point, **and** add zeroes when needed to the most significant end of the integer part a d to the least significant end of the fractional part.

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NEVER add a zero between the least significant integer digit and the radix point, or between the most significant fractional digit and the radix point..

Question 11:

(a) $2525.2524_8 = 555.554_{16}$
(b) $2735.6734_8 = 5DD.DDC_{16}$
(c) $001\ 000\ 001\ 000\ 001.000\ 001\ 000\ 001_2 = 1041.041_{16}$
(d) $001\ 001\ 001\ 000\ 001.001\ 001\ 000\ 001\ 001_2 = 1241.2412_{16}$
(e) $0001\ 0000\ 0001.0000\ 0001_2 = 401.002_8$
(f) $0001\ 0001\ 0000\ 0001.0001\ 0001_2 = 10401.042_8$
(g) $001\ 010\ 011\ 100\ 101.110\ 111\ 001\ 010\ 100_2 = 14E5.DCA8_{16}$
(h) $0001\ 0010\ 0011\ 0100\ 0101.0110\ 0111\ 0001\ 0010\ 0100_2 = 221505.316111_8$
(i) $1101\ 1110\ 1010\ 1101.1010\ 1100\ 1110_2 = 157255.5316_8$
(j) $1011\ 1010\ 1101.1011\ 1110\ 1110\ 1111_2 = 5655.575674_8$

Revised 15 Sep 98; Corrected 20 Sep 2000