CS-252 Discrete Structures
Department of Computer Science
James Madison University

Test #1
October 13, 2003
Time Allowed: 45 Minutes

Name **(PRINT)** Model Answer by Mohamed Aboutabl

Score **(Max 20 points)**

Attempt ALL questions. Put your answers in the provided space.

In questions 1 and 2 below, determine whether the proposition is TRUE or FALSE, and why so.

2 points 1. $\underbrace{1+1=3}_{F}$ if and only if $\underbrace{2+2=3}_{F}$. True since $(F \leftrightarrow F)$ is true

2 points 2. If $\underbrace{2+1=3}_{T}$, then $\underbrace{2=3-1}_{T}$. True since $(T \rightarrow T)$ is true

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3 points 3. Write the contrapositive, converse, and inverse of the following:
"If you try hard, then you will win."

Contrapositive: if (you will not win) then (you do not try hard)

Converse: if (you will win) then (you try hard)

Inverse: if (you do not try hard) then (you will not win)

In questions 4 and 5 below suppose the variable x represents students and y represents courses, and:
$U(y)$: y is an upper-level course $M(y)$: y is a math course $F(x)$: x is a freshman
$A(x)$: x is a part-time student $T(x,y)$: student x is taking course y.
Write each statement using these predicates and any needed quantifiers.

2 points 4. Every student is taking at least one course.

$\forall x \; \exists y \; T(x,y)$

2 points 5. There is a part-time student who is not taking any math course.

$\exists x \; \forall y \; \left(A(x) \wedge \left(M(y) \rightarrow \neg T(x,y)\right) \right)$

OR $\exists x,\; A(x) \wedge \left(\forall y,\; M(y) \rightarrow \neg T(x,y) \right)$

OR $\exists x, \forall y \;\; A(x) \wedge \left(T(x,y) \rightarrow \neg M(y) \right)$

2 points

6. In this questions $P(x,y)$ means "$x + 2y = xy$", where x and y are integers. Determine the truth value of the statement (and why so):

$$\exists y P(3,y).$$

True because $P(3,y): \ 3 + 2y = 3y \implies y = 3$

In questions 7-9 below suppose $A = \{a,b,c\}$. Mark the statement TRUE or FALSE. P(A) is the power set of A. Notice $P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

1 point 7. $\{b,c\} \in P(A)$. True

1 point 8. $\varnothing \subseteq A \times A$. True since ϕ is a subset of any set.

1 point 9. $\{\{a\}\} \subseteq P(A)$. True

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2 points

10. Suppose $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ where $g(x) = 2x + 1$ and $g \circ f(x) = 2x + 11$. Find the rule for f.

Let $y = f(x)$

$\therefore \underbrace{g \circ f(x)}_{\text{given}} = g(f(x))$, Let y denote $f(x)$

$2x + 11 = \underbrace{g(y)}$

$2x + 11 = 2 \cdot y + 1$, by definition of $g(\cdot)$

$\therefore y = \frac{2x + 10}{2} = x + 5$, by solving for y

$\therefore \boxed{f(x) = x + 5}$

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2 points

11. Prove or disprove(by a counterexample) : For all integers a,b,c, if $a \mid c$ and $b \mid c$, then $ab \mid c^2$.

True. Proof

Let a, b, c be arbitrary integers such that $a|c$ and $b|c$

$\therefore c = k \cdot a$ for some integer k

also $c = \ell \cdot b$ for some integer ℓ

Multiply both equations

$\therefore c^2 = (k \cdot \ell) \cdot ab \qquad \therefore ab \mid c^2$

My Best Wishes
Dr. Mohamed Aboutabl