Practice Questions for Chapters 1 and 2

In questions 1, 2 below, determine whether the proposition is TRUE or FALSE

T F **1.** If $1 < 0$, then $3 = 4$.

T F **2.** If $1 + 1 = 2$ or $1 + 1 = 3$, then $2 + 2 = 3$ and $2 + 2 = 4$.

3. Determine whether $(p \to q) \wedge (\neg p \to q) \equiv q$.

4. Write a proposition equivalent to $p \vee \neg q$ that uses only $p, q, \neg$ and the connective $\wedge$.

5. Prove that $p \to q$ and its converse are not logically equivalent.

6. Determine whether the following two propositions are logically equivalent: $p \to (\neg q \wedge r), \neg p \vee \neg(r \to q)$.

In questions 7,8 below, write the statement in the form "If ..., then ...."

7. To get a good grade it is necessary that you study.

8. You need to be registered in order to check out library books.

9. Write the contrapositive, converse, and inverse of the following: You sleep late if it is Saturday.

In questions 10, 11 below write the negation of the statement. (Don't write "It is not true that ....")

10. If it is rainy, then we go to the movies.

11. I will go to the play or read a book, but not both.

12. Using c for "it is cold" and r for "it is rainy", write "It is rainy if it is not cold" in symbols.

In question 13 below $P(x,y)$ means "$x + 2y = xy$", where x and y are integers. Determine the truth value of the statement.

T F **13.** $P(0,0)$.

In questions 14, 15 below suppose the variable x represents students and y represents courses, and:

$U(y)$: y is an upper-level course $M(y)$: y is a math course $F(x)$: x is a freshman

$B(x)$: x is a full-time student $T(x,y)$: student x is taking course y.

Write the statement using these predicates and any needed quantifiers.

14. All students are freshmen.

15. No math course is upper-level.

16. Explain why the negation of "Some students in my class use e-mail" is not "Some students in my class do not use e-mail".

17. Determine whether the following argument is valid:

$\therefore p \to r$

$\therefore q \to r$

$\therefore \neg(p \vee q)$

ThereFore : $\neg r$

18. Show that the premises "Every student in this class passed the first exam" and "Alvina is a student in this class" imply the conclusion "Alvina passed the first exam".

19. Consider the following theorem: If x is an odd integer, then $x + 2$ is odd. Give an indirect proof of this theorem.

20. Prove that the following is true for all positive integers n: n is even if and only if $3n^2 + 8$ is even.

21. Prove or disprove: For all real numbers x and y, $\lfloor x - y \rfloor = \lfloor x \rfloor - \lfloor y \rfloor$.

22. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a containment proof (that is, prove that the left side is a subset of the right side and that the right side is a subset of the left side).

23. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving an element table proof.

24. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a proof using logical equivalence.

25. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a Venn diagram proof.

26. Prove or disprove: if A, B, and C are sets, then $A - (B \cap C) = (A - B) \cap (A - C)$.

In questions 27-28, use a Venn diagram to determine which relationship, $\subseteq$, $=$, $\supseteq$, is true for the pair of sets.

27. $A \cup B,\ A \cup (B - A)$.

28. $A \cup (B \cap C),\ (A \cup B) \cap C$.

In questions 29, 30 below determine whether the given set is the power set of some set. If the set is a power set, give the set of which it is a power set.

29. $\{\varnothing,\{a\},\{\varnothing,a\}\}$.

30. $\{\varnothing,\{a\},\{\varnothing\},\{a,\varnothing\}\}$.

In questions 31-36 below suppose $A = \{1,2,3,4,5\}$. Mark the statement TRUE or FALSE.

T F **31.** $\{1\} \in P(A)$.

T F **32.** $\varnothing \subseteq A$.

T F **33.** $\{\varnothing\} \subseteq P(A)$.

T F **34.** $\varnothing \subseteq P(A)$.

T F **35.** $\{2,4\} \in A \times A$.

T F **36.** $\{\varnothing\} \in P(A)$.

In questions 37-41 below determine whether the rule describes a function with the given domain and codomain.

37. $f : \mathbf{N} \to \mathbf{N}$ where $f(n) = \sqrt{n}$.

38. $h : \mathbf{R} \to \mathbf{R}$ where $h(x) = \sqrt{x}$.

39. $g : \mathbf{N} \to \mathbf{N}$ where $g(n)$ = any integer $> n$.

40. $F : \mathbf{Z} \to \mathbf{R}$ where $F(x) = \dfrac{1}{x^2 - 5}$.

41. $f : \mathbf{R} \to \mathbf{R}$ where $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ x - 1 & \text{if } x \ge 4 \end{cases}$

42. Suppose $f : \mathbf{N} \to \mathbf{N}$ has the rule $f(n) = 4n^2 + 1$. Determine whether f is onto **N**.

43. Suppose $f : \mathbf{R} \to \mathbf{R}$ where $f(x) = \lfloor x/2 \rfloor$.
(a) Draw the graph of f.
(b) Is f 1-1?
(c) Is f onto **R**?

In questions 44 below find the inverse of the function f or else explain why the function has no inverse.

44. Suppose $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ where $g(x) = 2x + 1$ and $g \circ f(x) = 2x + 11$. Find the rule for f.

45. Prove or disprove: For all integers a,b,c,d, if $a \mid b$ and $c \mid d$, then $(a + c) \mid (b + d)$.

46. Prove or disprove: For all integers a,b,c,d, if $a \mid b$ and $c \mid d$, then $(ac) \mid (b + d)$.

47. Prove or disprove: For all integers a,b,c, if $a \mid c$ and $b \mid c$, then $ab \mid c^2$.

48. Find the prime factorization of 510,510.

49. List all positive integers less than 30 that are relatively prime to 20.

50. Find –88 **mod 13.**

51. Find three integers m such that $13 \equiv 7 \pmod{m}$.

In questions 52-55 below determine whether each of the following "theorems" is true or false. Assume that a, b, c, d, and m are integers with $m > 1$.

T F **52.** If $a \equiv b \pmod{m}$, and $a \equiv c \pmod{m}$, then $a \equiv b + c \pmod{m}$.

T F **53.** If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv b + d \pmod{m}$.

T F **54.** If $a \equiv b \pmod{m}$, then $2a \equiv 2b \pmod{2m}$.

T F **55.** If $a \equiv b \pmod{2m}$, then $a \equiv b \pmod{m}$.

56. Encrypt the message NEED HELP by translating the letters into numbers, applying the encryption function $f(p) = (p + 3)$ **mod** 26, and then translating the numbers back into letters.

57. What sequence of pseudorandom numbers is generated using the pure multiplicative generator $x_{n+1} = 3x_n \textbf{ mod } 11$ with seed $x_0 = 2$?

58. Suppose that a computer has only the memory locations 0,1,2,...,19. Use the hashing function h where $h(x) = (x + 5)$ **mod** 20 to determine the memory locations in which 57, 32, and 97 are stored.

59. Explain why $f(x) = (2x + 3)$ **mod** 26 would not be a good coding function.

60. Convert $(204)_{10}$ to base 2.

61. Convert $(11101)_2$ to base 10.

62. Convert $(BC1)_{16}$ to base 2.

63. Take any three-digit integer, reverse its digits, and subtract. For example, $742 - 247 = 495$. The difference is divisible by 9. Prove that this must happen for all three-digit numbers abc.

64. Use the Euclidean Algorithm to find gcd(900,140).

Answer Key

1. True
2. False
3. Both truth tables are identical:

p	q	$(p \to q) \wedge (\neg p \to q)$	q
T	T	T	T
T	F	F	F
F	T	T	T
F	F	F	F

4. $\neg(\neg p \wedge q)$.
5. Truth values differ when p is true and q is False
6. Yes.
7. If you don't study, then you don't get a good grade (equivalently, if you get a good grade, then you study).
8. If you are not registered, then you cannot check out library books (equivalently, if you check out library books, then you are registered).
9. Contrapositive: If you do not sleep late, then it is not Saturday. Converse: If you sleep late, then it is Saturday. Inverse: If it is not Saturday, then you do not sleep late.
10. It is rainy and we do not go to the movies.
11. I will go to the play and read a book, or I will not go to the play and not read a book.
12. $\neg c \to r$.
13. True
14. $\forall x F(x)$.
15. $\forall y(M(y) \to \neg U(y))$.
16. Both statements can be true at the same time.
17. Not valid: p false, q false, r True
18. Universal instantiation.
19. Suppose $x + 2 = 2k$. Therefore $x = 2k - 2 = 2(k - 1)$, which is even.
20. If n is even, then $n = 2k$. Therefore $3n^2 + 8 = 3(2k)^2 + 8 = 12k^2 + 8 = 2(6k^2 + 4)$, which is even. If n is odd, then $n = 2k + 1$. Therefore $3n^2 + 8 = 3(2k + 1)^2 + 8 = 12k^2 + 12k + 11 = 2(6k^2 + 6k + 5) + 1$, which is odd.
21. False: $x = 2$ $y = 1/2$.
22. $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$: Let $x \in \overline{A \cap B}$. $\therefore$ $x \notin A \cap B$, $\therefore$ $x \notin A$ or $x \notin B$, $\therefore$ $x \in \overline{A}$ or $x \in \overline{B}$, $\therefore$ $x \in \overline{A} \cup \overline{B}$. Reversing the steps shows that $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$.
23. The columns for $\overline{A \cap B}$ and $\overline{A} \cup \overline{B}$ match: each entry is 0 if and only if A and B have the value 1.

24. $\overline{A \cap B} = \{x \mid x \in \overline{A \cap B}\}$

$= \{x \mid x \notin A \cap B\}$

$= \{x \mid \neg(x \in A \cap B)\}$

$= \{x \mid \neg(x \in A \wedge x \in B)\}$

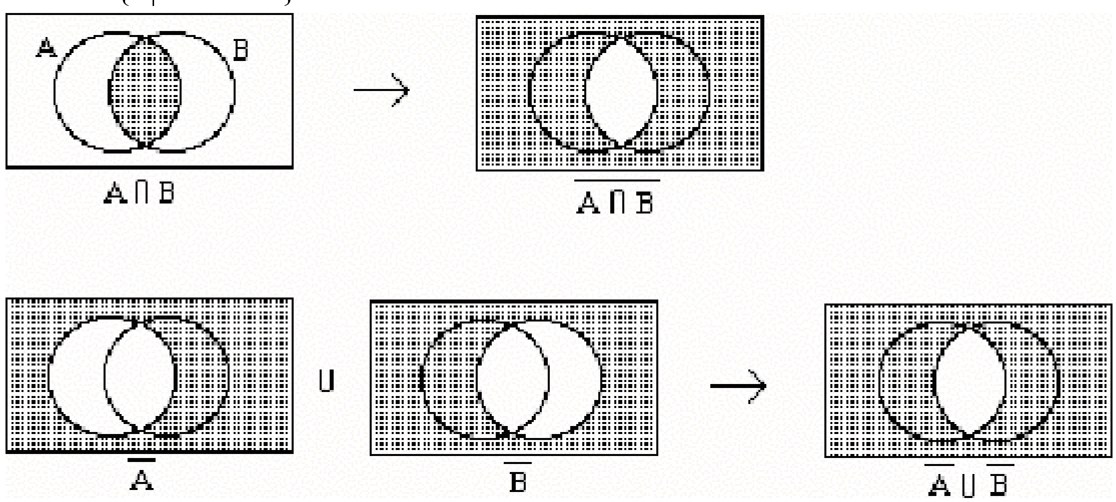
$= \{x \mid \neg(x \in A) \vee \neg(x \in B)\}$

$= \{x \mid x \notin A \vee x \notin B\}$

$= \{x \mid x \in \overline{A} \vee x \in \overline{B}\}$

$= \{x \mid x \in \overline{A} \cup \overline{B}\} = \overline{A} \cup \overline{B}$

25.



26. False. For example, let $A = \{1,2\}$, $B = \{1\}$, $C = \{2\}$.

27. =.

28. $\supseteq$.

29. No, it lacks $\{\varnothing\}$.

30. Yes, $\{\{a,\varnothing\}\}$.

31. True

32. True

33. True

34. True

35. False

36. False

37. Not a function; $f(2)$ is not an integer.

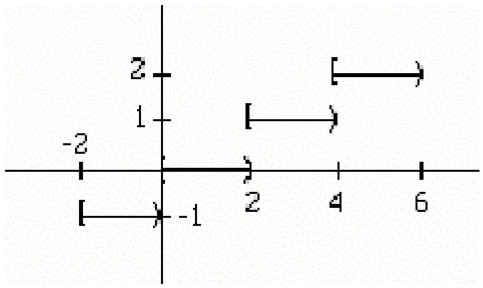
38. Function.

39. Not a function; $g(1)$ has more than one value.

40. Function.

41. Not a function; $f(3)$ not defined.

42. No.

43. (a) 

(b) No.

(c) No.

44. $f(x) = x + 5$.

45. False: $a = b = c = 1$, $d = 2$.

46. False: $a = b = 2, c = d = 1$.

47. True: If $c = ak$ and $c = bl$, then $c^2 = ab(kl)$, so $ab \mid c^2$.

48. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17$.

49. 1,3,7,9,11,13,17,19,21,23,27,29.

50. 3.

51. 2,3,6.

52. False

53. False

54. True

55. True

56. Encrypted form: QHHG KHOS.

57. The sequence 2,6,7,10,8 repeats.

58. 2,17,3.

59. f is not $1-1$ ($f(0) = f(13)$), and hence f^{-1} is not a function.

60. 1100 1100.

61. 29.

62. 1011 1100 0001.

63. $abc - cba = 100a + 10b + c - (100c + 10b + a) = 99a - 99c = 9(11a - 11c)$. Therefore $9 \mid abc - cba$.

64. 20.