CS-252 Discrete Structures
Department of Computer Science
James Madison University

Test #2
November 21, 2003
Time Allowed: 45 Minutes

Name (PRINT) MODEL ANSWER

Section No. Circle **1** **2**

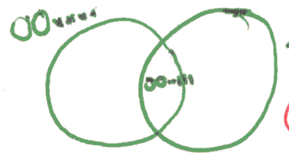
Score (Max 20 points) By Dr. Mohamed Aboutabl

Attempt ALL questions. Put your answers in the provided space.

2 points

1. How many bit strings of length seven either begin with two 0s or end with three 1s?

Ex # 40 pg. 312



$A = \{00.....\}$ $\quad B = \{....111\}$

$\frac{3}{4}$ $|A| = 2^5 = 32$ $\quad |B| = 2^4 = 16$

$\frac{3}{4}$ $|A \cap B| = 2^2 = 4$

$\therefore |A \cup B| = |A| + |B| - |A \cap B| = 32 + 16 - 4 = 44$ $\quad \frac{1}{2}$

4 points

2. Thirteen people on a softball team show up for a game

Ex # 26 page 325

a) How many ways are there to choose ten players to take the field?

$1\frac{1}{2}$ $\quad \binom{13}{10} = 286 = \dfrac{13!}{10!\ \ 3!}$

b) How many ways are there to assign the ten positions by selecting players from the 13 people who show up?

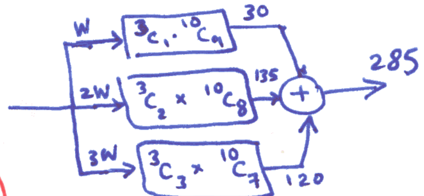
$1\frac{1}{2}$ $\quad {}^{13}P_{10} = 1,037,836,800 = \dfrac{13!}{(13-10)!}$

c) Of the 13 people who show up, three are women. How many ways are there to choose ten players to take the field if at least one of these players must be a woman?

1 $\quad = \binom{13}{10}$ - # of ways to select a team with no women

$= \binom{13}{10} - \underbrace{\binom{10}{10}}_{\text{All men}} = 286 - 1 = 285$

OR



Notice we need at least (not exactly) one woman.

i.e. one W or two W's or three W's

3 points 3. Prove that if n and k are positive integers, then $\binom{n+1}{k} = \frac{n+1}{k}\binom{n}{k-1}$

Ex 23 page 333

$$R.H.S = \frac{n+1}{k}\binom{n}{k-1} = \frac{n+1}{k} \cdot \frac{n!}{(k-1)!\ (n-k+1)!}$$

$$= \frac{(n+1)(n!)}{k\cdot(k-1)!\ (n+1-k)!} = \frac{(n+1)!}{k!\ (n+1-k)!} = \binom{n+1}{k}$$

$= R.H.S.$

5 points 4. Use the Principle of Mathematical Induction to prove that $2n + 3 \leq 2^n$ for all $n \geq 4$.

Ex 29 page 254

$P(n)$: $2n+3 \leq 2^n \quad \forall\, n \geq 4$

Basis Step @ $n=4$

$2*4+3 \ \overset{?}{\leq}\ 2^4$

$11 \leq 16$ $\quad \therefore P(4)$ is true

Inductive Step To Show $P(k) \rightarrow P(k+1)$

where $P(k)$: $2k+3 \leq 2^k \quad \forall\, k \geq 4$

$P(k+1)$: $2(k+1)+3 \leq 2^{k+1}$

Consider $2(k+1)+3 = (2k+3)+2$

$< 2^k + 2$, by $P(k)$

$< 2^k + 2^k$, since $2 < 2^k \quad \forall\, k \geq 4$

$\therefore\ 2(k+1)+3 < 2^{k+1}$, since $2^k + 2^k = 2 \cdot 2^k = 2^{k+1}$

$\therefore P(k+1)$ is true.

2 points

Ex 8 page 333

5. What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$?

$\binom{17}{8}(3x)^8 \cdot (2y)^9$

∴ $\boxed{\binom{17}{8} * 3^8 * 2^9}$ is the coefficient of $x^8.y^9$

$= 24,310 * 6,561 * 512 = 81,662,929,920$

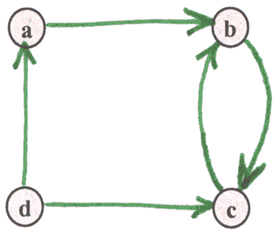
$=$ which is the same as $\boxed{\binom{17}{9} * 3^8 * 2^9}$

4 points

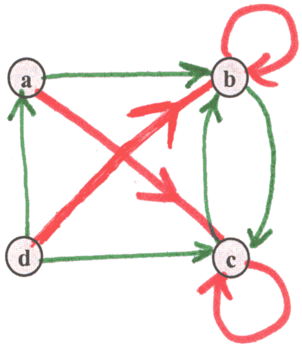
6. The following is a relation on the set { a , b , c , d }:

R = { (a , b) , (b , c) , (c , b) , (d , c) , (d , a) }

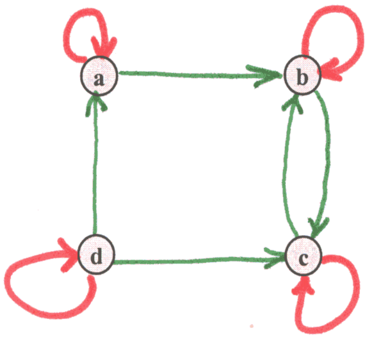
i) Draw the directed graph of this relation by completing the figure below



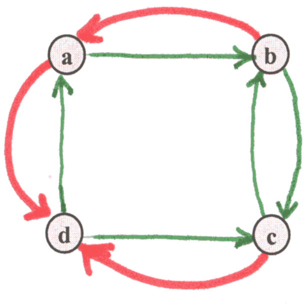
ii) Now, draw the transitive closure of R



iii) and the reflexive closure of R



iv) and the symmetric closure of R



Notice that any closure contains the original relation in addition to possibly more ordered pairs (i.e. arcs)

My Best Wishes
Dr. Mohamed Aboutabl