Combinatorics Identities

1. The number of r-permutations of a set with $n \geq 0$ distinct elements is

$$ {}^{n}P_{r} = P(n,r) \quad = \quad n \bullet (n-1) \bullet (n-2) \cdots\cdots \bullet (n-r+1) \quad = \quad \frac{n!}{(n-r)!} \quad , \quad 0 \leq r \leq n $$

2. The number of r-combinations of a set with $n \geq 0$ distinct elements is

$$ {}^{n}C_{r} \quad = \quad C(n,r) \quad = \quad \frac{n!}{r!\,(n-r)!} \quad , \quad 0 \leq r \leq n $$

3. The Binomial Theorem: Let x and y be variables, and let n be a nonnegative integer. Then:

$$ \begin{aligned} (x+y)^{n} &= \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^{j} \\ &= \binom{n}{0} x^{n} + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^{2} + \cdots + \binom{n}{n-1} x\, y^{n-1} + \binom{n}{n} y^{n} \end{aligned} $$

4. Let n be a nonnegative integer. Then

$$ \sum_{j=0}^{n} \binom{n}{j} \quad = \quad 2^{n} $$

5. Let n be a nonnegative integer. Then

$$ \sum_{j=0}^{n} (-1)^{j} \binom{n}{j} \quad = \quad 0 $$

6. Let n be a nonnegative integer. Then

$$ \sum_{j=0}^{n} 2^{j} \binom{n}{j} \quad = \quad 3^{n} $$

7. Pascal's Identity: Let n and k be a positive integers with $n \geq k$. Then

$$ \binom{n+1}{k} \quad = \quad \binom{n}{k-1} \quad + \quad \binom{n}{k} $$

8. Let n be a nonnegative integer. Then

$$ \binom{2n}{n} \quad = \quad \sum_{k=0}^{n} \binom{n}{k}^{2} $$

9. Let n and r be nonnegative integers with $r \leq n$. Then:

$$ \binom{n+1}{r+1} \quad = \quad \sum_{j=r}^{n} \binom{j}{r} $$