1.7 SET OPERATIONS

Geometric figures were used by John Venn (1834-1923) to illustrate the effect of various operations on sets called the ***universal set*** or the ***domain of discourse***.

Remark: Not all set operations can be represented by Venn diagrams.

VENN DIAGRAMS

DEF: In a generic ***Venn diagram*** for a subset S of a fixed universal set U, the universal set is represented by a rectangular region in the plane and the set S is represented by a subregion.

Fig 1.7.1 Set S is shaded.

DYADIC SET OPERATIONS

DEF: The ***union*** of sets S and T is the set containing every object that is either in S or in T.
NOTATION: $S \cup T$.

Fig 1.7.2 Union $S \cup T$ is shaded.

DEF: The ***intersection*** of sets S and T is the set containing every object that is in both S and T.
NOTATION: $S \cap T$.

Fig 1.7.3 Intersection $S \cap T$ is shaded.

DEF: The ***difference*** of sets S and T is the set containing every object that is in S but not in T.
NOTATION: $S - T$.

Fig 1.7.4 Difference $S - T$ is shaded.

Example 1.7.1:

$S = \{1, 3, 5, 7, 9\} \quad T = \{2, 3, 5, 7\}$. Then

$S \cup T = \{1, 2, 3, 5, 7, 9\}$.

$S \cap T = \{3, 5, 7\}$.

$S - T = \{1, 9\}$.

Example 1.7.2: Cartesian product (which is dyadic) is not representable by a Venn diagram.

MONADIC SET OPERATIONS

DEF: The ***complement*** of a set S is the set $U - S$, where U is the universal set. NOTATION: $\overline{S}$.

Fig 1.7.5 Complement $\overline{S}$ is shaded.

Example 1.7.3: These monadic operations are not readily representable by Venn diagrams.

$S \to \{S\}$ (***enbracement***)

$S \to P(S)$ (***empowerment***)

TERMINOLOGY NOTE: These two (original) names have excellent mnemonicity. The mildly frivolous character may deter their widespread adoption.

SET IDENTITIES

Various set equivalences have earned the honorific appelation ***identity***. Many of them are analogous to the logical equivalences of §1.2.

Example 1.7.4: The Double Negation Law $\neg\neg p \Leftrightarrow p$ has the following set-theoretic analogy:

DEF: ***Double Complementation Law:***

$$\overline{\overline{S}} = S.$$

Example 1.7.5: The tautology $p \vee \neg p$ (called the Law of the Excluded Middle) converts to the equivalence

$$p \vee \neg p \Leftrightarrow T,$$

which has the following set-theoretic analogy:

$$S \cup \overline{S} = U.$$

AVOIDING BOREDOM

Example 1.7.6: Table 1 of §1.7 (de Morgan, associativity, etc.) is good for self-study, but not for exhaustive classroom presentation.

CONFIRMING IDENTITIES with VENN DIAGRAMS

Example 1.7.7: $\cap$ distributes over $\cup$.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Example 1.7.8: $\cup$ distributes over $\cap$.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$