1.7 SET OPERATIONS

Geometric figures were used by John Venn (1834-1923) to illustrate the effect of various operations on sets called the universal set or the domain of discourse.

Remark: Not all set operations can be represented by Venn diagrams.

VENN DIAGRAMS

DEF: In a generic Venn diagram for a subset $S$ of a fixed universal set $U$, the universal set is represented by a rectangular region in the plane and the set $S$ is represented by a subregion.

Fig 1.7.1 Set $S$ is shaded.
DYADIC SET OPERATIONS

DEF: The **union** of sets $S$ and $T$ is the set containing every object that is either in $S$ or in $T$.

NOTATION: $S \cup T$.

**Fig 1.7.2** Union $S \cup T$ is shaded.

DEF: The **intersection** of sets $S$ and $T$ is the set containing every object that is in both $S$ and $T$.

NOTATION: $S \cap T$.

**Fig 1.7.3** Intersection $S \cap T$ is shaded.
DEF: The **difference** of sets $S$ and $T$ is the set containing every object that is in $S$ but not in $T$.

**NOTATION:** $S - T$.

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**Fig 1.7.4** Difference $S - T$ is shaded.

**Example 1.7.1:**

$S = \{1, 3, 5, 7, 9\} \quad T = \{2, 3, 5, 7\}$. Then

$S \cup T = \{1, 2, 3, 5, 7, 9\}$.

$S \cap T = \{3, 5, 7\}$.

$S - T = \{1, 9\}$.

**Example 1.7.2:** Cartesian product (which is dyadic) is not representable by a Venn diagram.
MONADIC SET OPERATIONS

DEF: The *complement* of a set \( S \) is the set \( U - S \), where \( U \) is the universal set. NOTATION: \( \bar{S} \).

Fig 1.7.5  Complement \( \bar{S} \) is shaded.

Example 1.7.3: These monadic operations are not readily representable by Venn diagrams.

\[ S \rightarrow \{S\} \text{ (*enbracement*)} \]

\[ S \rightarrow P(S) \text{ (*empowerment*)} \]

TERMINOLOGY NOTE: These two (original) names have excellent mnemonicity. The mildly frivolous character may deter their widespread adoption.
SET IDENTITIES

Various set equivalences have earned the honorific appellation identity. Many of them are analogous to the logical equivalences of §1.2.

Example 1.7.4: \(\neg\neg p \iff p\) has the following set-theoretic analogy:

**DEF:** *Double Complementation Law:*

\[\overline{\overline{S}} = S.\]

Example 1.7.5: The tautology \(p \lor \neg p\) (called the Law of the Excluded Middle) converts to the equivalence

\[p \lor \neg p \iff T,\]

which has the following set-theoretic analogy:

\[S \cup \overline{S} = U.\]

AVOIDING BOREDOM

Example 1.7.6: Table 1 of §1.7 (de Morgan, associativity, etc.) is good for self-study, but not for exhaustive classroom presentation.
CONFIRMING IDENTITIES with VENN DIAGRAMS

Example 1.7.7: $\cap$ distributes over $\cup$.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Example 1.7.8: $\cup$ distributes over $\cap$.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$