1.4 NESTED QUANTIFIERS

Example 1.4.1: Every sophomore owns a computer or has a friend in the junior class who owns a computer.

Domains S and J are the sophomores and the juniors. Predicates $C(u)$ and $F(v, w)$ mean that u owns a computer and that w is a friend of v.

$$(\forall x \in S)\big[C(x) \vee (\exists y \in J)[F(x, y) \wedge C(y)]\big].$$

DISAMBIGUATION: Specify the domain when not evident from context. Use brackets to identify scope of quantifiers.

TRANSPOSING QUANTIFIERS

Be careful about transposing different kinds of quantifiers.

$(\forall x)(\exists y)[x^2 \leq y]$ is true.

$(\exists y)(\forall x)[x^2 \leq y]$ is false.

However, you can safely transpose two quantifiers of the same kind.

RECALL NEGATION with QUANTIFIERS

p: There exists some input data for which this program will crash.

$\neg p$: No matter what input data you supply to this program, it will not crash.

Rule 1: $\neg(\exists x)[P(x)] \Leftrightarrow (\forall x)[\neg P(x)]$

Rule 2: $\neg(\forall x)[P(x)] \Leftrightarrow (\exists x)[\neg P(x)]$

CLASSROOM EXERCISE

Write the negation of this statement

$$(\forall x)(\exists y)[x^2 \leq y]$$

so that no negation ($\neg$) appears to the left of a quantifier.

$\neg(\forall x)(\exists y)[x^2 \leq y] =$

OPTIONAL CLASSROOM EXERCISE

An exercise about varying the subdomain from within the set of all people.

B(x,y) : y is the brother of x (predicate)

Specify a subdomain (maximal, if possible) in which each of the following assertions is TRUE.

1. $(\forall x)(\forall y)[B(x,y) \rightarrow B(y,x)]$.

For any two persons Bill(x) and George(y), if George(y) is a brother of Bill(x), then Bill(x) is the brother of George(y).

2. $(\exists x)(\forall y)[B(x,y) \rightarrow B(y,x)]$.

There is a person who is a brother to each of his brothers.

3. $(\forall x)(\exists y)[B(x,y) \rightarrow B(y,x)]$.

Every person has a brother to whom that person is also a brother.

4. $(\exists x)(\exists y)[B(x,y) \rightarrow B(y,x)]$.

There exist two persons, Bill (x) and George (y), such that if George is Bill's brother, then Bill is George's brother.