1.2 PROPOSITIONAL EQUIVALENCES

Propositional equivalences occur in mathematical proofs. They are also useful in simplifying loop-exit conditions in computer programs.

DEF: Two propositional forms on the same variables are ***(logically) equivalent*** if they have the same result column in their truth tables.
NOTATION: $F \Leftrightarrow G$.

DISAMBIGUATION: The biconditional $\leftrightarrow$ is an operator. Logical equivalence $\Leftrightarrow$ is a relation on propositions.

Example 1.2.1: $\neg p \vee q \Leftrightarrow p \rightarrow q$

CONTRAPOSITIVE, etc.

DEF: The ***contrapositive*** of the propositional form $p \to q$ is the form $\neg q \to \neg p$.

Proposition 1.2.1. *The contrapositive of* $p \to q$ *is logically equivalent to* $p \to q$.

Proof:

Example 1.2.2:

conditional $p \to q$: If it is sunny, then you can find me at the beach.

contrapositive $\neg q \to \neg p$: If you can't find me at the beach, then it is not sunny.

DEF: The ***converse*** of the propositional form $p \to q$ is the form $q \to p$.

DEF: The ***inverse*** of the propositional form $p \to q$ is the form $\neg p \to \neg q$.

Example 1.2.3:

conditional $p \to q$: If it is sunny, then you can find me at the beach.

converse $q \to p$: If you can find me at the beach, then it is sunny.

inverse $\neg p \to \neg q$: If it is not sunny, then you can't find me at the beach.

Proposition 1.2.2. *The converse and the inverse are equivalent to each other, but not to the original conditional.*

Proof: Left to the reader. $\diamondsuit$

CATEGORIES of PROPOSITIONAL FORMS

DEF: A ***tautology*** is a propositional form that is always true, no matter what truth values are assigned to its variables.

DEF: A ***self-contradiction*** is a propositional form that is always false, no matter what truth values are assigned to its variables.

DEF: A ***contingency*** is a propositional form that is neither a tautology nor a contradiction.

DISAMBIGUATION: The word "contradiction" means two propositions with opposite truth values. See Methods of Proof in §1.5.

Proposition 1.2.3. *A propositional form is a tautology iff it is equivalent to the constant T.*

Proof: This is simply a rephrasing. ♢

Proposition 1.2.4. *A propositional form is a self-contradiction iff it is equivalent to the constant F.*

Proof: This is a rephrasing. ♢

LAWS of LOGIC

Various logical equivalences and tautologies have earned the honorific appelation ***law***.

DEF: ***Double Negation Law:*** $\neg\neg p \Leftrightarrow p$.

DEF: ***Law of the Excluded Middle:*** $p \vee \neg p$.

AVOIDING BOREDOM

First Law of Good Pedagogy: Boredom does not help anyone to learn.

Example 1.2.4: Table 5 of §1.2 (de Morgan, associativity, etc.) is excellent for self-study, but not for exhaustive classroom presentation.