1.2 PROPOSITIONAL EQUIVALENCES

Propositional equivalences occur in mathematical proofs. They are also useful in simplifying loop-exit conditions in computer programs.

**DEF:** Two propositional forms on the same variables are *(logically) equivalent* if they have the same result column in their truth tables.

**NOTATION:** $F \iff G$.

**DISAMBIGUATION:** The biconditional $\iff$ is an operator. Logical equivalence $\iff$ is a relation on propositions.

**Example 1.2.1:** $\neg p \lor q \iff p \to q$
CONTRAPOSITIVE, etc.

DEF: The *contrapositive* of the propositional form \( p \rightarrow q \) is the form \( \neg q \rightarrow \neg p \).

**Proposition 1.2.1.** *The contrapositive of* \( p \rightarrow q \) *is logically equivalent to* \( p \rightarrow q \).

**Proof:**

**Example 1.2.2:**

- **conditional** \( p \rightarrow q \): If it is sunny, then you can find me at the beach.
- **contrapositive** \( \neg q \rightarrow \neg p \): If you can’t find me at the beach, then it is not sunny.
DEF: The **converse** of the propositional form \( p \rightarrow q \) is the form \( q \rightarrow p \).

DEF: The **inverse** of the propositional form \( p \rightarrow q \) is the form \( \neg p \rightarrow \neg q \).

**Example 1.2.3:**

conditional \( p \rightarrow q \): If it is sunny, then you can find me at the beach.

converse \( q \rightarrow p \): If you can find me at the beach, then it is sunny.

inverse \( \neg p \rightarrow \neg q \): If it is not sunny, then you can’t find me at the beach.

**Proposition 1.2.2.** *The converse and the inverse are equivalent to each other, but not to the original conditional.*

**Proof:** Left to the reader.
CATEGORIES of PROPOSITIONAL FORMS

DEF: A \textbf{tautology} is a propositional form that is always true, no matter what truth values are assigned to its variables.

DEF: A \textbf{self-contradiction} is a propositional form that is always false, no matter what truth values are assigned to its variables.

DEF: A \textbf{contingency} is a propositional form that is neither a tautology nor a contradiction.

DISAMBIGUATION: The word “contradiction” means two propositions with opposite truth values. See Methods of Proof in §1.5.

Proposition 1.2.3. A propositional form is a tautology iff it is equivalent to the constant $T$.

Proof: This is simply a rephrasing. \hfill \Diamond

Proposition 1.2.4. A propositional form is a self-contradiction iff it is equivalent to the constant $F$.

Proof: This is a rephrasing. \hfill \Diamond
**LAWS of LOGIC**

Various logical equivalences and tautologies have earned the honorific appellation *law*.

**DEF:** *Double Negation Law:* \( \neg \neg p \iff p \).

**DEF:** *Law of the Excluded Middle:* \( p \lor \neg p \).

**AVOIDING BOREDOM**

*First Law of Good Pedagogy:* Boredom does not help anyone to learn.

*Example 1.2.4:* Table 5 of §1.2 (de Morgan, associativity, etc.) is excellent for self-study, but not for exhaustive classroom presentation.