

HW# 7.5

⑧ Two strings x and y are related by R , i.e. $(x, y) \in R$, iff $x = \boxed{x_1 x_2 x_3 A}$ and $y = \boxed{y_1 y_2 y_3 A}$, where A is some string of bits.

* it is clear that every string of bits is related with itself i.e. $\forall x, (x, x) \in R$. so, R is reflexive

* if $(x, y) \in R$ then $(x = x_1 x_2 x_3 A$ and $y = y_1 y_2 y_3 A)$ therefore, according to the definition of R , $(y, x) \in R$

so, R is symmetric

* if $(x, y) \in R$ and $(y, z) \in R$ then

$$x = \boxed{x_1 x_2 x_3 A} \quad \text{and} \quad y = \boxed{y_1 y_2 y_3 A} \quad \text{and} \quad z = \boxed{z_1 z_2 z_3 A}$$

by examining the strings x and z we conclude that $(x, z) \in R$

so, R is transitive

from the above, R is an equivalence relation.

18. a) This is not an equivalence relation, since it is not symmetric.

b) This is an equivalence relation; one equivalence class consists of the first and third elements, and the other consists of the second and fourth elements.

c) This is an equivalence relation; one equivalence class consists of the first, second, and third elements, and the other consists of the fourth element.

38. We need to show that every equivalence class consisting of people living in the same county (or parish) and same state is contained in an equivalence class of all people living in the same state. This is clear. The equivalence class of all people living in county c in state s is a subset of the set of people living in state s .