HW # 10.2

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4. a) We need to write all the terms that have $\overline{x}$ in them. Thus the answer is $\overline{x}\,y\,z + \overline{x}\,y\,\overline{z} + \overline{x}\,\overline{y}\,z + \overline{x}\,\overline{y}\,\overline{z}$.

b) We need to write all the terms that include either $\overline{x}$ or $\overline{y}$. Thus the answer is $x\,\overline{y}\,z + x\,\overline{y}\,\overline{z} + \overline{x}\,y\,z + \overline{x}\,y\,\overline{z} + \overline{x}\,\overline{y}\,z + \overline{x}\,\overline{y}\,\overline{z}$.

c) We need to include all the terms that have both $\overline{x}$ and $\overline{y}$. Thus the answer is $\overline{x}\,\overline{y}\,z + \overline{x}\,\overline{y}\,\overline{z}$.

d) We need to include all the terms that have at least one of $\overline{x}$, $\overline{y}$, and $\overline{z}$. This is all the terms except $x\,y\,z$, so the answer is $x\,y\,\overline{z} + x\,\overline{y}\,z + x\,\overline{y}\,\overline{z} + \overline{x}\,y\,z + \overline{x}\,y\,\overline{z} + \overline{x}\,\overline{y}\,z + \overline{x}\,\overline{y}\,\overline{z}$.

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12. We need to use De Morgan's law to replace each occurrence of $s+t$ by $\overline{(\overline{s}\,\overline{t})}$, simplifying by use of the double complement law if possible.

a) $(x+y)+z = \overline{(\overline{(x+y)}\,\overline{z})} = \overline{(\overline{x}\,\overline{y}\,\overline{z})}$ b) $x+\overline{y}\,(\overline{x}+z) = \overline{(\overline{x}\,\overline{(\overline{y}\,\overline{(\overline{x}+z)})})} = \overline{(\overline{x}\,\overline{(\overline{y}\,\overline{\overline{(x\,\overline{z})}})})}$

c) In this case we can just apply De Morgan's law directly, to obtain $\overline{x}\,\overline{\overline{y}} = \overline{x}\,y$.

d) The second factor is changed in a manner similar to part (a). Thus the answer is $\overline{x}\overline{(\overline{x}\,y\,z)}$.

14. a) We use the definition of $\mid$. If $x=1$, then $x \mid x = 0$; and if $x=0$, then $x \mid x = 1$. These are precisely the corresponding values of $\overline{x}$.

b) We can construct a table to look at all four cases, as follows. Since the fourth and fifth columns are equal, the expressions are equivalent.

x	y	$x \mid y$	$(x \mid y) \mid (x \mid y)$	xy
1	1	0	1	1
1	0	1	0	0
0	1	1	0	0
0	0	1	0	0

c) We can construct a table to look at all four cases, as follows. Since the fifth and sixth columns are equal, the expressions are equivalent.

x	y	$x \mid x$	$y \mid y$	$(x \mid x) \mid (y \mid y)$	$x+y$
1	1	0	0	1	1
1	0	0	1	1	1
0	1	1	0	1	1
0	0	1	1	0	0