6. Under the hypotheses, we have \( c = as \) and \( d = bt \) for some \( s \) and \( t \). Multiplying we obtain \( cd = ab(st) \), which means that \( ab \mid cd \), as desired.

8. The numbers 19, 101, 107, and 113 are prime, as we can verify by trial division. The numbers 27 = 3³ and 93 = 3 · 31 are not prime.

10. In each case we can carry out the arithmetic on a calculator.
   a) Since \( 8 \cdot 5 = 40 \) and \( 44 - 40 = 4 \), we have quotient \( 44 \ \text{div} \ 8 = 5 \) and remainder \( 44 \ \text{mod} \ 8 = 4 \).
   b) Since \( 21 \cdot 37 = 777 \), we have quotient \( 777 \ \text{div} \ 21 = 37 \) and remainder \( 777 \ \text{mod} \ 21 = 0 \).
   c) As above, we can compute \( 123 \ \text{div} \ 19 = 6 \) and \( 123 \ \text{mod} \ 19 = 9 \). However, since the dividend is negative and the remainder is nonzero, the quotient is \( -(6 + 1) = -7 \) and the remainder is \( 19 - 9 = 10 \). To check that \( -123 \ \text{div} \ 19 = -7 \) and \( -123 \ \text{mod} \ 19 = 10 \), we note that \( -123 = (-7)(19) + 10 \).
   d) Since \( 1 \ \text{div} \ 23 = 0 \) and \( 1 \ \text{mod} \ 23 = 1 \), we have \( -1 \ \text{div} \ 23 = -1 \) and \( -1 \ \text{mod} \ 23 = 22 \).
   e) Since \( 2002 \ \text{div} \ 87 = 23 \) and \( 2002 \ \text{mod} \ 87 = 1 \), we have \( -2002 \ \text{div} \ 87 = -24 \) and \( 2002 \ \text{mod} \ 87 = 86 \).
   f) Clearly \( 0 \ \text{div} \ 17 = 0 \) and \( 0 \ \text{mod} \ 17 = 0 \).
   g) We have \( 1234567 \ \text{div} \ 1001 = 1233 \) and \( 1234567 \ \text{mod} \ 1001 = 334 \).
   h) Since \( 100 \ \text{div} \ 101 = 0 \) and \( 100 \ \text{mod} \ 101 = 100 \), we have \( -100 \ \text{div} \ 101 = -1 \) and \( -100 \ \text{mod} \ 101 = 1 \).

18. Since these numbers are small, the easiest approach is to find the prime factorization of each number and look for any common prime factors.
   a) Since \( 21 = 3 \cdot 7 \), \( 34 = 2 \cdot 17 \), and \( 55 = 5 \cdot 11 \), these are pairwise relatively prime.
   b) Since \( 85 = 5 \cdot 17 \), these are not pairwise relatively prime.
   c) Since \( 25 = 5^2 \), \( 41 \) is prime, \( 49 = 7^2 \), and \( 64 = 2^6 \), these are pairwise relatively prime.
   d) Since \( 17, 19, \) and \( 23 \) are prime and \( 18 = 2 \cdot 3^2 \), these are pairwise relatively prime.

32. We have \( 1000 = 2^3 \cdot 5^3 \) and \( 625 = 5^4 \), so \( \gcd(1000, 625) = 5^3 = 125 \), and \( \text{lcm}(1000, 625) = 2^3 \cdot 5^4 = 5000 \). As expected, \( 125 \cdot 5000 = 625000 = 1000 \cdot 625 \).

42. From \( a \equiv b \pmod{m} \) we know that \( b = a + sm \) for some integer \( s \). Similarly, \( d = c + tm \). Subtracting, we have \( b - d = (a - c) + (s - t)m \), which means that \( a - c \equiv b - d \pmod{m} \).

54. We just need to “subtract 3” from each letter. For example, E goes down to B, and B goes down to Y.
   a) BLUE JEANS     b) TEST TODAY     c) EAT DIM SUM

56. We have the 2x2 matrix A =...