HW 2.4

6. Under the hypotheses, we have $c = as$ and $d = bt$ for some s and t. Multiplying we obtain $cd = ab(st)$, which means that $ab \mid cd$, as desired.

8. The numbers 19, 101, 107, and 113 are prime, as we can verify by trial division. The numbers $27 = 3^3$ and $93 = 3 \cdot 31$ are not prime.

10. In each case we can carry out the arithmetic on a calculator.
a) Since $8 \cdot 5 = 40$ and $44 - 40 = 4$, we have quotient $44 \textbf{ div } 8 = 5$ and remainder $44 \textbf{ mod } 8 = 4$.
b) Since $21 \cdot 37 = 777$, we have quotient $777 \textbf{ div } 21 = 37$ and remainder $777 \textbf{ mod } 21 = 0$.
c) As above, we can compute $123 \textbf{ div } 19 = 6$ and $123 \textbf{ mod } 19 = 9$. However, since the dividend is negative and the remainder is nonzero, the quotient is $-(6+1) = -7$ and the remainder is $19 - 9 = 10$. To check that $-123 \textbf{ div } 19 = -7$ and $-123 \textbf{ mod } 19 = 10$, we note that $-123 = (-7)(19) + 10$.
d) Since $1 \textbf{ div } 23 = 0$ and $1 \textbf{ mod } 23 = 1$, we have $-1 \textbf{ div } 23 = -1$ and $-1 \textbf{ mod } 23 = 22$.
e) Since $2002 \textbf{ div } 87 = 23$ and $2002 \textbf{ mod } 87 = 1$, we have $-2002 \textbf{ div } 87 = -24$ and $2002 \textbf{ mod } 87 = 86$.
f) Clearly $0 \textbf{ div } 17 = 0$ and $0 \textbf{ mod } 17 = 0$.
g) We have $1234567 \textbf{ div } 1001 = 1233$ and $1234567 \textbf{ mod } 1001 = 334$.
h) Since $100 \textbf{ div } 101 = 0$ and $100 \textbf{ mod } 101 = 100$, we have $-100 \textbf{ div } 101 = -1$ and $-100 \textbf{ mod } 101 = 1$.

18. Since these numbers are small, the easiest approach is to find the prime factorization of each number and look for any common prime factors.
a) Since $21 = 3 \cdot 7$, $34 = 2 \cdot 17$, and $55 = 5 \cdot 11$, these are pairwise relatively prime.
b) Since $85 = 5 \cdot 17$, these are not pairwise relatively prime.
c) Since $25 = 5^2$, 41 is prime, $49 = 7^2$, and $64 = 2^6$, these are pairwise relatively prime.
d) Since 17, 19, and 23 are prime and $18 = 2 \cdot 3^2$, these are pairwise relatively prime.

32. We have $1000 = 2^3 \cdot 5^3$ and $625 = 5^4$, so $\gcd(1000, 625) = 5^3 = 125$, and $\operatorname{lcm}(1000, 625) = 2^3 \cdot 5^4 = 5000$. As expected, $125 \cdot 5000 = 625000 = 1000 \cdot 625$.

42. From $a \equiv b \pmod{m}$ we know that $b = a + sm$ for some integer s. Similarly, $d = c + tm$. Subtracting, we have $b - d = (a - c) + (s - t)m$, which means that $a - c \equiv b - d \pmod{m}$.

54. We just need to "subtract 3" from each letter. For example, E goes down to B, and B goes down to Y.
a) BLUE JEANS b) TEST TODAY c) EAT DIM SUM