HW 1.8

8. We simply round up or down in each case.

a) 1 b) 2 c) -1 d) 0 e) 3 f) -2 g) $\lfloor \frac{1}{2} + 1 \rfloor = \lfloor \frac{3}{2} \rfloor = 1$

h) $\lceil 0 + 1 + \frac{1}{2} \rceil = \lceil \frac{3}{2} \rceil = 2$

18. If we can find an inverse, the function is a bijection. Otherwise we must explain why the function is not on-to-one or not onto.

a) This is a bijection since the inverse function is $f^{-1}(x) = (4 - x)/3$.

b) This is not one-to-one since $f(17) = f(-17)$, for instance. It is also not onto, since the range is the interval $(-\infty, 7]$. For example, 42548 is not in the range.

c) This function is a bijection, but not from $\mathbf{R}$ to $\mathbf{R}$. To see that the domain and range are not $\mathbf{R}$, note that $x = -2$ is not in the domain, and $x = 1$ is not in the range. On the other hand, f is a bijection from $\mathbf{R} - \{-2\}$ to $\mathbf{R} - \{1\}$, since its inverse is $f^{-1}(x) = (1 - 2x)/(x - 1)$.

d) It is clear that this continuous function is increasing throughout its entire domain ($\mathbf{R}$) and it takes on both arbitrarily large values and arbitrarily small (large negative) ones. So it is a bijection. Its inverse is clearly $f^{-1}(x) = \sqrt[5]{x - 1}$.

28. We have $(f \circ g)(x) = f(g(x)) = f(x + 2) = (x + 2)^2 + 1 = x^2 + 4x + 5$, whereas $(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = x^2 + 1 + 2 = x^2 + 3$. Note that they are not equal.

62. This follows immediately from the definition. We want to show that $((f \circ g) \circ (g^{-1} \circ f^{-1}))(z) = z$ for all $z \in Z$ and that $((g^{-1} \circ f^{-1}) \circ (f \circ g))(x) = x$ for all $x \in X$. For the first we have

$$\begin{aligned}((f \circ g) \circ (g^{-1} \circ f^{-1}))(z) &= (f \circ g)((g^{-1} \circ f^{-1})(z)) \\ &= (f \circ g)(g^{-1}(f^{-1}(z))) \\ &= f(g(g^{-1}(f^{-1}(z)))) \\ &= f(f^{-1}(z)) = z.\end{aligned}$$