HW 1.7

11. This exercise asks for a proof of one of De Morgan's laws for sets. The primary way to show that two sets are equal is to show that each is a subset of the other. In other words, to show that $X = Y$, we must show that whenever $x \in X$, it follows that $x \in Y$, and that whenever $x \in Y$, it follows that $x \in X$. Exercises 5–7 could also have been done this way, but it was easier in those cases to reduce the problems to the corresponding problems of logic. Here, too, we can reduce the problem to logic and invoke De Morgan's law for logic, but this problem requests specific proof techniques.

a) This proof is similar to the proof of the dual property, given in Example 10. Suppose $x \in \overline{A \cup B}$. Then $x \notin A \cup B$, which means that x is in neither A nor B. In other words, $x \notin A$ and $x \notin B$. This is equivalent to saying that $x \in \overline{A}$ and $x \in \overline{B}$. Therefore $x \in \overline{A} \cap \overline{B}$, as desired. Conversely, if $x \in \overline{A} \cap \overline{B}$, then $x \in \overline{A}$ and $x \in \overline{B}$. This means $x \notin A$ and $x \notin B$, so x cannot be in the union of A and B. Since $x \notin A \cup B$, we conclude that $x \in \overline{A \cup B}$, as desired.

b) The following membership table gives the desired equality, since columns four and seven are identical.

A	B	$A \cup B$	$\overline{A \cup B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cap \overline{B}$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

18. First suppose x is in the left-hand side. Then x must be in A but in neither B nor C. Thus $x \in A - C$, but $x \notin B - C$, so x is in the right-hand side. Next suppose that x is in the right-hand side. Thus x must be in $A - C$ and not in $B - C$. The first of these implies that $x \in A$ and $x \notin C$. But now it must also be the case that $x \notin B$, since otherwise we would have $x \in B - C$. Thus we have shown that x is in A but in neither B nor C, which implies that x is in the left-hand side.

28. There are precisely two ways that an item can be in either A or B but not both. It can be in A but not B (which is equivalent to saying that it is in $A - B$), or it can be in B but not A (which is equivalent to saying that it is in $B - A$). Thus an element is in $A \oplus B$ if and only if it is in $(A - B) \cup (B - A)$.

49. **a)** The multiplicity of a in the union is the maximum of 3 and 2, the multiplicities of a in A and B. Since the maximum is 3, we find that a occurs with multiplicity 3 in the union. Working similarly with b, c (which appears with multiplicity 0 in B), and d (which appears with multiplicity 0 in A), we find that $A \cup B = \{3 \cdot a, 3 \cdot b, 1 \cdot c, 4 \cdot d\}$.

b) This is similar to part (a), with "maximum" replaced by "minimum." Thus $A \cap B = \{2 \cdot a, 2 \cdot b\}$. (In particular, c and d appear with multiplicity 0—i.e., do not appear—in the intersection.)

c) In this case we subtract multiplicities, but never go below 0. Thus the answer is $\{1 \cdot a, 1 \cdot c\}$.

d) Similar to part (c) (subtraction in the opposite order); the answer is $\{1 \cdot b, 4 \cdot d\}$.

e) We add multiplicities here, to get $\{5 \cdot a, 5 \cdot b, 1 \cdot c, 4 \cdot d\}$.