HW 1.5

8. a) If we use modus tollens starting from the back, then we conclude that I am not sore. Another application of modus tollens then tells us that I did not play hockey.

b) We really can't conclude anything specific here.

c) By universal instantiation, we conclude from the first implication by modus ponens that dragonflies have six legs, and we conclude by modus tollens that spiders are not insects. We could say using existential generalization that, for example, there exists a non-six-legged creature that eats a six-legged creature, and that there exists a non-insect that eats an insect.

d) We can apply universal instantiation to the implication and conclude that if Homer (respectively, Maggie) is a student, then he (she) has an Internet account. Now modus tollens tells us that Homer is not a student. There are no conclusions to be drawn about Maggie.

e) The first implication is that if x is healthy to eat, then x does not taste good. Universal instantiation and modus ponens therefore tell us that tofu does not taste good. The third sentence says that if you eat x, then x tastes good. Therefore the fourth hypothesis already follows (by modus tollens) from the first three. No conclusions can be drawn about cheeseburgers from these statements.

f) By disjunctive syllogism, the first two hypotheses allow us to conclude that I am hallucinating. Therefore by modus ponens we know that I see elephants running down the road.

38. We need to prove two things, since this is an "if and only if" statement. First let us prove directly that if n is even then $7n+4$ is even. Since n is even, it can be written as $2k$ for some integer k. Then $7n+4 = 14k+4 = 2(7k+2)$. This is 2 times an integer, so it is even, as desired. Next we give an indirect proof that if $7n+4$ is even then n is even. So suppose that n is not even, i.e., that n is odd. Then n can be written as $2k+1$ for some integer k. Thus $7n+4 = 14k+11 = 2(7k+5)+1$. This is 1 more than 2 times an integer, so it is odd. That completes the indirect proof.