HW 1.4

10. a) $\forall x F(x, \text{Fred})$ b) $\forall y F(\text{Evelyn}, y)$ c) $\forall x \exists y F(x, y)$ d) $\neg\exists x \forall y F(x, y)$ e) $\forall y \exists x F(x, y)$
f) $\neg\exists x[F(x, \text{Fred}) \wedge F(x, \text{Jerry})]$
g) $\exists y_1 \exists y_2[F(\text{Nancy}, y_1) \wedge F(\text{Nancy}, y_2) \wedge y_1 \neq y_2 \wedge \forall y(F(\text{Nancy}, y) \rightarrow (y = y_1 \vee y = y_2))]$
h) $\exists y[\forall x F(x, y) \wedge \forall z(\forall x F(x, z) \rightarrow z = y)]$ i) $\neg\exists x F(x, x)$
j) $\exists x \exists y[x \neq y \wedge F(x, y) \wedge \forall z((F(x, z) \wedge z \neq x) \rightarrow z = y)]$ (We do not assume that this sentence is asserting that this person can fool her/himself.)

18. a) $\forall f\,(H(f) \rightarrow \exists c\, A(c))$, where $A(x)$ means that console x is accessible, and $H(x)$ means that fault condition x is happening
b) $(\forall u \exists m\,(A(m) \wedge S(u, m))) \rightarrow \forall u\, R(u)$, where $A(x)$ means that the archive contains message x, $S(x, y)$ means that user x sent message y, and $R(x)$ means that the e-mail address of user x can be retrieved
c) $(\forall b \exists m\, D(m, b)) \leftrightarrow \exists p \neg C(p)$, where $D(x, y)$ means that mechanism x can detect breach y, and $C(x)$ means that process x has been compromised
d) $\forall x \forall y\,(x \neq y \rightarrow \exists p \exists q\,(p \neq q \wedge C(p, x, y) \wedge C(q, x, y)))$, where $C(p, x, y)$ means that path p connects endpoint x to endpoint y
e) $\forall x\,((\forall u\, K(x, u)) \leftrightarrow x = \text{SysAdm})$, where $K(x, y)$ means that person x knows the password of user y

36. a) In English, the negation is "Some student in this class does not like mathematics." With the obvious propositional function, this is $\exists x \neg L(x)$.
b) In English, the negation is "Every student in this class has seen a computer." With the obvious propositional function, this is $\forall x S(x)$.
c) In English, the negation is "For every student in this class, there is a mathematics course that this student has not taken." With the obvious propositional function, this is $\forall x \exists c \neg T(x, c)$.
d) As in Exercise 15f, let $P(z, y)$ be "Room z is in building y," and let $Q(x, z)$ be "Student x has been in room z." Then the original statement is $\exists x \forall y \exists z(P(z, y) \wedge Q(x, z))$. To form the negation, we change all the quantifiers and put the negation on the inside, then apply De Morgan's law. The negation is therefore $\forall x \exists y \forall z(\neg P(z, y) \vee \neg Q(x, z))$, which is also equivalent to $\forall x \exists y \forall z(P(z, y) \rightarrow \neg Q(x, z))$. In English, this could be read, "For every student there is a building such that for every room in that building, the student has not been in that room."