

### Composition of Relations is Associative.

To show:  $(R \circ S) \circ T = R \circ (S \circ T)$

Proof:

$$\begin{aligned}(R \circ S) \circ T &= \left\{ (a,b) \mid \exists c, \quad (a,c) \in T \quad \wedge \quad \underbrace{(c,b) \in (R \circ S)} \right\} \\ &= \left\{ (a,b) \mid \exists c, \exists d, \quad \underbrace{(a,c) \in T \quad \wedge \quad (c,d) \in S} \quad \wedge \quad (d,b) \in R \right\} \\ &= \left\{ (a,b) \mid \exists d, \quad \underbrace{(a,d) \in (S \circ T) \quad \wedge \quad (d,b) \in R} \right\} \\ &= R \circ (S \circ T)\end{aligned}$$