

16. *Proof (by mathematical induction):*

**The formula is true for  $n = 0$  :** The formula holds for  $n = 0$  because  $\prod_{i=0}^0 \left( \frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) = \frac{1}{1} \cdot \frac{1}{2} = \frac{1}{2}$  and  $\frac{1}{(2 \cdot 0 + 2)!} = \frac{1}{2}$  also.

**If the formula is true for  $n = k$  then it is true for  $n = k + 1$  :** Suppose  $\prod_{i=0}^k \left( \frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) = \frac{1}{(2k+2)!}$  for some integer  $k \geq 0$ . We must show that  $\prod_{i=0}^{k+1} \left( \frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) = \frac{1}{(2 \cdot (k+1) + 2)!}$ , or, equivalently,  $\prod_{i=0}^{k+1} \left( \frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) = \frac{1}{((2k+4)!}$ . But by the laws of algebra and substitution from the inductive hypothesis,

$$\begin{aligned} \prod_{i=0}^{k+1} \left( \frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) &= \prod_{i=0}^k \left( \frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) \cdot \left( \frac{1}{2(k+1)+1} \cdot \frac{1}{2(k+1)+2} \right) \\ &= \left( \frac{1}{(2k+2)!} \right) \cdot \left( \frac{1}{2(k+1)+1} \cdot \frac{1}{2(k+1)+2} \right) \\ &= \frac{1}{(2k+2)!} \cdot \frac{1}{2k+3} \cdot \frac{1}{2k+4} \\ &= \frac{1}{(2k+4)!} \end{aligned}$$

[as was to be shown].