32. *Proof*: Suppose m and n are any *[particular but arbitrarily chosen]* integers such that $n - m$ is even. *[We must show that $n^3 - m^3$ is even.]* Note that $n^3 - m^3 = (n-m)(n^2 + nm + m^2)$. Now $n - m$ is even by supposition and $n^2 + nm + m^2$ is an integer (being a sum of products of integers). Thus $(n-m)(n^2 + nm + m^2)$ is the product of an even integer and an integer, and so, by exercise 28, it is even. Hence, by substitution, $n^3 - m^3$ is even *[as was to be shown]*.

(Alternatively, the given statement can be proved by direct use of the definition of even, without reference to exercise 28.)

39. *Proof*: Suppose a and b are any nonnegative real numbers. Then

$$\sqrt{a} = \text{that unique nonnegative real number u such that } u^2 \text{ equals } a$$

and

$$\sqrt{b} = \text{that unique nonnegative real number v such that } v^2 \text{ equals } b$$

By substitution and the laws of exponents, $ab = u^2v^2 = (uv)^2$. So uv is that unique nonnegative real number such that $(uv)^2 = ab$. Hence $\sqrt{ab} = uv = \sqrt{a}\sqrt{b}$.